OPEN PROBLEMS IN THE WILD MCKAY CORRESPONDENCE

TAKEHIKO YASUDA

Contents

The aim of this note is to provide a list of open problems related to the wild McKay correspondence, a generalization of the McKay correspondence to positive and mixed characteristics. Around the summer of 2011, I started a study on this subject. Since then, there have been considerable progress and I have wrote several papers, two of which are joint papers with Melanie Wood; they are [24, 19, 16, 21, 20, 15] in chronological order depending on the dates when they appeared on arXiv. By now, a number of open problems have accumulated. Some of them have been already asked, but scattered in these papers. Therefore it would be meaningful to collect them here in one place, in particular, for students or young researchers looking for problems.

The wild McKay correspondence is a delightful interaction of the geometry of singularities (in particular, from the perspective of birational geometry) and the arithmetic problems such as counting extensions of a local field and counting rational points over a finite field. Therefore it would shed new light on both the birational geometry and such arithmetic problems. In the birational geometry, after the recent achievement in characteristic zero [5], there is a trend toward positive characteristics. However, it seems still hard to treat low characteristics relative to dimension. What the wild McKay correspondence mainly concerns is such a situation of low characteristics and may bring a hope that the birational geometry as we have in characteristic zero will be eventually carried out in arbitrary characteristic to some extent. As for the number theory, the wild McKay correspondence

Date: June 9, 2015.

provides a new way of counting extensions of a local field. Namely they had been counted with weights determined by classical invariants such as discriminants. The wild McKay correspondence produces plenty of new weights of geometric origin, the study whose arithmetic meaning is awaited.

Problems presented in this note are not be exhaustive, but rather chosen according to the author's subjective viewpoint. For instance, we do not discuss recent progress on resolution of wild quotient surface singularities (see [7] and references therein). The McKay correspondence over a global field, discussed in [17, 18], is also beyond the scope of the present note.

1. A brief overview

Before discussing open problems, we give here a brief overview of several versions of the wild McKay correspondence, some of which are still conjectural.

1.1. **The linear and absolute case.** For several reasons, it is natural and sometimes necessary to work over a complete discrete valuation ring: we call it the *relative* setting. However we first consider the *absolute* setting, that is, work over a perfect field. Algebraic geometers would be mainly interested in this case. We also restrict ourselves to linear actions on affine spaces, because non-linear ones should be treated in the relative setting.

There are many versions of the McKay correspondence today. The one which we generalize is the McKay correspondence in terms of stringy invariants proved by Batyrev [2]. In the simplest form, his result is stated as follows. Let G be a finite subgroup of $\mathrm{SL}_d(\mathbb{C})$ and X the associated quotient variety $\mathbb{C}^d/G$. The result is the equality in a certain modification of the Grothendieck ring of varieties $K_0(\mathrm{Var}_{\mathbb{C}})$.

$$M_{\mathrm{st}}(X) = \sum_{g \in \mathrm{Conj}(G)} \mathbb{L}^{\mathrm{age}(g)}. \tag{1.1}$$

Here $M_{\mathrm{st}}(X)$ is the stringy motif (or stringy motivic invariant, which sounds more precise) of X, $\mathbb{L}$ is the class of an affine line $\mathbb{A}^1_{\mathbb{C}}$, $\mathrm{Conj}(G)$ is the set of conjugacy classes of G and $\mathrm{age}(g)$ is the *age* of g, which is determined by eigenvalues of g and depends only on the conjugacy class (see [8]). In general, for a log terminal variety X, if $f : Y \to X$ is a log resolution and $K_{Y/X} = \sum_{i \in I} a_i E_i$, $a_i \in \mathbb{Q}$, is the relative canonical divisor with E_i prime exceptional divisors, then the stringy motif $M_{\mathrm{st}}(X)$ is given by the formula

$$M_{\mathrm{st}}(X) = \sum_{J \subset I} [E_J^\circ] \prod \frac{\mathbb{L} - 1}{\mathbb{L}^{1+a_i} - 1}. \tag{1.2}$$

In particular, if $Y \to X$ is a crepant resolution, then $M_{\mathrm{st}}(X) = [Y]$. In fact, he used stringy E-functions rather than motivic invariants. But it is rather straightforward to translate his result to motivic invariants. His proof was somehow computational. A more conceptual approach was found by Denef–Loeser [4]. Equality (1.1), in fact, holds also for a small finite subgroup of $G \subset \mathrm{GL}_d(\mathbb{C})$, that is a finite subgroup without pseudo-reflection. When the group is not small, we only need to replace $M_{\mathrm{st}}(X)$ with the stringy motif $M_{\mathrm{st}}(X, D)$ of the log pair (X, D), where D is the branch divisor on X with standard coefficients.[1] See [23].

[1]The divisor should be chosen so that the morphism $\mathbb{C}^d \to (X, D)$ is crepant. In general, by a morphism $(Y, E) \to (X, D)$ of pairs, we simply mean a morphism $Y \to X$ of underlying varieties.

Next consider a finite subgroup $G \subset \mathrm{GL}_d(k)$ with k a perfect field of arbitrary characteristic. For brevity, suppose that it has no pseudo-reflection. The quotient variety is normal and $\mathbb{Q}$-Gorenstein. Hence one can define the stringy motif $M_{\mathrm{st}}(X)$, not by using a resolution, but by the integral expression over the arc space of X given in [4]. On the other hand, the right hand side of (1.1) needs a considerable change. The idea is to replace $\mathrm{Conj}(G)$ with the conjectural moduli space $G\text{-}\mathrm{Cov}(k((t)))$ of G-covers (étale G-torsors) of the formal punctured disk $\mathrm{Spec}\, k((t))$. This should be defined over k and for each k-algebra A, its A-points corresponds to G-torsors over $\mathrm{Spec}\, A((t))$. In precise, this should be a stack not algebraic but close to being algebraic. By an abuse of terminology and notation, we say that L is a G-cover of $k((t))$ is $\mathrm{Spec}\, L$ is a G-cover of $\mathrm{Spec}\, k((t))$ and write $L \in G\text{-}\mathrm{Cov}(k((t)))$. To each $L \in G\text{-}\mathrm{Cov}(k((t)))$, we define $\mathbf{v}(L) \in \mathbb{Q}$ as follows. Let $\mathcal{O}_L$ be the integral closure of $k[[t]]$ in L, which has a natural G-action. Consider two G-actions on $\mathcal{O}_L^{\oplus d}$, then one is the diagonal action induced from the G-action on $\mathcal{O}_L$ and the other is the one induced from the embedding $G \subset \mathrm{GL}_d(k) \subset \mathrm{GL}_d(\mathcal{O}_L)$.[2] We define the *tuning submodule* $\Xi_L \subset \mathcal{O}_L^{\oplus d}$ to be the subset of those elements where the two actions coincide. We then put

$$\mathbf{v}(L) := \frac{1}{\sharp G}\mathrm{colengh}(\mathcal{O}_L \cdot \Xi_L \subset \mathcal{O}_L^{\oplus d}).$$

This defines a function $\mathbf{v} : G\text{-}\mathrm{Cov}(k((t))) \to \mathbb{Q}$ and conjecturally have constructible subsets as fibers.

Conjecture 1.1. *We have*

$$M_{\mathrm{st}}(X) = \int_{G\text{-}\mathrm{Cov}(k((t)))} \mathbb{L}^{d-\mathbf{v}}.$$

The integral on the right hand is defined as

$$\sum_{r \in \mathbb{Q}} [\mathbf{v}^{-1}(r)]\mathbb{L}^r.$$

If $k = \mathbb{C}$, then there is a one-to-one correspondence between $\mathrm{Conj}(G)$ and $G\text{-}\mathrm{Cov}(\mathbb{C}((t)))$, and the functions age and $d - \mathbf{v}$ correspond to each other by this correspondence. In this way, we see that this is indeed a generalization of Batyrev's McKay correspondence (1.1).

The conjecture holds for $G = \mathbb{Z}/p\mathbb{Z}$ [24].

1.2. **The relative setting.** There are a few advantages in the relative setting, in which we work over a complete discrete valuation ring $\mathcal{O}_K$ with perfect residue field k. Firstly it is more general than the absolute setting: to switch from the absolute to the relative, we merely take the scalar extension (base change) from k to $\mathcal{O}_K = k[[t]]$. In the relative setting, we can take, in particular, the p-adic integer ring $\mathbb{Z}_p$ or a finite extension of it as $\mathcal{O}_K$, which is more interesting for number-theorists. Secondly the untwisting technique, which was the key for proofs of main results in [4, 20], is carried out only in the relative setting. Thirdly, to formulate

We identify a normal $\mathbb{Q}$-Gorenstein variety X with the pair $(X, 0)$. We say that a generically étale morphism $f : (Y, E) \to (X, D)$ is *crepant* if $f^*(K_X + D) = K_Y + E$.

[2] Here we need to be careful about how to define G-actions on $\mathbb{A}^d_k$ and its coordinate ring; on which do matrices in G acts, $\mathbb{A}^d_k$ or the coordinate ring?; from which does it act, left or right? These are not unified through my papers [16, 15, 24, 20, 19, 21]. In this note, we ignore this problem, avoiding detailed description of actions.

the wild McKay correspondence for non-linear actions, we need the untwisting technique, and hence need to work in the relative setting.

Let K be a complete discrete valuation field with perfect residue field k and let $\mathcal{O}_K$ be its integer ring (valuation ring). For instance, $K = k((t))$ for a perfect field k or K is a finite extension of $\mathbb{Q}_p$. We now suppose that G is a finite subgroup of $\mathrm{GL}_d(\mathcal{O}_K)$. We denote by $\mathcal{X}$ the quotient scheme $\mathbb{A}^d_{\mathcal{O}_K}/G$. We used the script $\mathcal{X}$ rather than the roman X, indicating that it is a scheme over $\mathcal{O}_K$ rather than k. We can define $M_{\mathrm{st}}(\mathcal{X})$ as a motivic integral on the space of arcs $\operatorname{Spec} \mathcal{O}_K \to \mathcal{X}$.[3] If there is a log resolution $f : \mathcal{Y} \to \mathcal{X}$, then we can express $M_{\mathrm{st}}(\mathcal{X})$ in a way similar to (1.2), but a little more involved. For brevity, suppose that the quotient morphism $\mathbb{A}^d_{\mathcal{O}_K} \to \mathcal{X}$ is étale in codimension one. Let $G\text{-}\mathrm{Cov}(K)$ be the conjectural moduli space defined over k of G-covers of $\operatorname{Spec} K$.[4] The function $\mathbf{v} : G\text{-}\mathrm{Cov}(K) \to \mathbb{Q}$ can be defined in the same way as in the absolute setting.

Conjecture 1.2. *We have*

$$M_{\mathrm{st}}(\mathcal{X}) = \int_{G\text{-}\mathrm{Cov}(K)} \mathbb{L}^{d-\mathbf{v}}.$$

Again, if $\mathbb{A}^d_{\mathcal{O}_K} \to \mathcal{X}$ is not étale in codimension one, then we need to replace $M_{\mathrm{st}}(\mathcal{X})$ with $M_{\mathrm{st}}(\mathcal{X}, D)$, where D is a $\mathbb{Q}$-divisor on $\mathcal{X}$ such that $\mathbb{A}^d_{\mathcal{O}_K} \to (\mathcal{X}, D)$ is crepant (the pull-back of $K_{\mathcal{X}/\mathcal{O}_K} + D$ is $K_{\mathbb{A}_{d_{\mathcal{O}_K/\mathcal{O}_K}}}$).

1.3. **The untwisting technique and the non-linear case.** We now review the untwisting technique (developed in [4, 19, 21]) and generalize the last conjecture to non-linear actions by using it. We continue to work over the ring $\mathcal{O}_K$ as above. Let $\mathcal{V} := \mathbb{A}^d_{\mathcal{O}_K}$. For each $L \in G\text{-}\mathrm{Cov}(K)$, we can construct another affine space $\mathcal{V}^{|L|} \cong \mathbb{A}^d_{\mathcal{O}_K}$ over $\mathcal{O}_K$ in a canonical manner. It is isomorphic to $\mathcal{V}^{|L|}$, but not canonically. In precise, $\mathcal{V}^{|L|}$ is defined so that its coordinate ring is the symmetric algebra of the dual $\mathrm{Hom}_{\mathcal{O}_K}(\Xi_L, \mathcal{O}_K)$ of the tuning module Ξ_L. We define $\mathcal{V}^{\langle L \rangle} := \mathcal{V}^{|L|} \otimes_{\mathcal{O}_K} \mathcal{O}_L \cong \mathbb{A}^d_{\mathcal{O}_L}$. There exists the canonical commutative diagram:

$$\begin{tikzcd} & \mathcal{V}^{\langle L \rangle} \arrow[dl] \arrow[dr] & \\ \mathcal{V} \arrow[dr] & & \mathcal{V}^{|L|} \arrow[dl] \\ & \mathcal{X} & \end{tikzcd}$$

The key fact in the untwisting is that the diagram induces a one-to-one correspondence between G-equivariant $\mathcal{O}_L$-points of $\mathcal{V}$ and $\mathcal{O}_K$-points of $\mathcal{V}^{|L|}$.

$$\mathcal{V}(\mathcal{O}_L)^G \leftrightarrow \mathcal{V}(\mathcal{O}_K)$$

If $\mathbf{v}(L) \neq 0$, then the morphism $\mathcal{V}^{|L|} \to \mathcal{X}$ contracts the closed subscheme $\mathcal{V}^{|L|}_k := \mathcal{V}^{|L|} \otimes_{\mathcal{O}_K} k \cong \mathbb{A}^d_k$ into a lower dimensional subvariety and makes the pair $(\mathcal{V}^{|L|}, -\mathbf{v}(L) \cdot$

[3] In precise, we need to use the Greenberg functor. See [12] for motivic integration over varieties defined over $\mathcal{O}_K$.

[4] To define the moduli functor precisely, we would need Witt rings. However, at least, k-points of $G\text{-}\mathrm{Cov}(K)$ should corresponds to G-covers of $\operatorname{Spec} K$.

$\mathcal{V}_k^{|L|}$) crepant over $\mathcal{X}$, or the pair $(\mathcal{X}, D)$ crepant to $\mathcal{V}$ if G is not small. We note that $\mathbb{L}^{d-\mathbf{v}(L)} = M_{\mathrm{st}}(\mathcal{V}^{|L|}, -\mathbf{v}(L) \cdot \mathcal{V}_k^{|L|})$.

Let us consider a normal affine $\mathcal{O}_K$-variety $\mathcal{W}$ and a $\mathbb{Q}$-divisor E on it such that $K_{\mathcal{W}/\mathcal{O}_K} + E$ is $\mathbb{Q}$-Cartier. Suppose that a finite group G acts on $\mathcal{W}$ faithfully and that E is preserved by the action. Let $\mathcal{Y} := \mathcal{W}/G$. There exists a unique $\mathbb{Q}$-divisor D on $\mathcal{Y}$ such that $(\mathcal{W}, E) \to (\mathcal{Y}, D)$ is crepant. The invariant $M_{\mathrm{st}}(\mathcal{Y}, D)$ is one side of the equality we are formulating. To get the other side, we fix an affine space $\mathcal{V} = \mathbb{A}^d_{\mathcal{O}_K}$ with a faithful G-action and a G-equivariant immersion $\mathcal{W} \to \mathcal{V}$. Such an immersion always exists. For each $L \in G\text{-Cov}(K)$, there exists the corresponding subvariety $\mathcal{W}^{|L|}$ of $\mathcal{V}^{|L|}$: it is defined as the preimage of the image of $\mathcal{W} \to \mathcal{X} = \mathcal{V}/G$. Let $\mathcal{W}^{|L|,\nu}$ be its normalization. There exists a unique $\mathbb{Q}$-divisor E_L on it such that the natural map $(\mathcal{W}^{|L|,\nu}, E_L) \to (\mathcal{Y}, D)$ is crepant. Let $\mathrm{Aut}(L)$ be the automorphism group of the G-cover $\operatorname{Spec} L \to \operatorname{Spec} K$. If H is the stabilizer of a connected component of $\operatorname{Spec} L$, then $\mathrm{Aut}(L)$ is isomorphic to $C_G(H)^{\mathrm{op}}$, the opposite group of the centralizer of H. There is a natural $\mathrm{Aut}(L)$-action on the pair $(\mathcal{W}^{|L|,\nu}, E_L)$. We can consider the quotient $M_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L)/\mathrm{Aut}(L)$.[5]

Conjecture 1.3. *We have*

$$M_{\mathrm{st}}(\mathcal{Y}, D) = \int_{G\text{-Cov}(K)} \frac{M_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L)}{\mathrm{Aut}(L)}.$$

Considering the stringy motif *along a constructible subset*, we can make this even more general. For a pair $(\mathcal{Z}, F)$ and a constructible subset $B \subset \mathcal{Z}_k = \mathcal{Z} \otimes_{\mathcal{O}_K} k$, we define $M_{\mathrm{st}}(\mathcal{Z}, F)_B$ to be the motivic integral with the same integrand as in the definition of $M_{\mathrm{st}}(\mathcal{Z}, F)$ and the set of arcs $\operatorname{Spec} \mathcal{O}_K \to \mathcal{Z}$ sending the closed point of $\operatorname{Spec} \mathcal{O}_K$ into B as the domain of integral. Let $C \subset \mathcal{W}_k$ be a constructible subset which is stable under the G-action, and $\overline{C}$ its image in $\mathcal{Y}_k$ and C_L the preimage of $\overline{C}$ in $\mathcal{W}_k^{|L|,\nu}$.

Conjecture 1.4 ([21])**.** *We have*

$$M_{\mathrm{st}}(\mathcal{Y}, D)_{\overline{C}} = \int_{G\text{-Cov}(K)} \frac{M_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L)_{C_L}}{\mathrm{Aut}(L)}.$$

1.4. **The point counting realization.** The main obstacle to proving the conjectures mentioned above is the lack of moduli spaces such as G-Cov(K) (see Section 9). It would need quite an effort to get it over. However, if we take their point count realization, this obstacle will disappear.

We now suppose that k is finite. For a k-variety X, we denote by $\sharp X$ the number of k-points of X. In the same way as defining M_{st} from M, we can define the string point count $\sharp_{\mathrm{st}}(\mathcal{X}, D) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ of the pair of a normal $\mathcal{O}_K$-variety $\mathcal{X}$ and a $\mathbb{Q}$-divisor D with $K_{\mathcal{X}/\mathcal{O}_K} + D$ $\mathbb{Q}$-Cartier. We can do it by replacing the motivic integration by the p-adic integration. By a slight abuse of notation, we denote the set (not the moduli space) of G-covers of K again by G-Cov(K).

[5] The quotient is taken in the following sense: if Z is a k-variety with an action of a finite group H, then $[Z]/H := [Z/H]$. In a few examples computed in [21], we had $M_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L) = M_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L)/\mathrm{Aut}(L)$ and taking the quotient did not change the invariant. We need to distinguish $M_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L)/\mathrm{Aut}(L)$ with the stringy invariant of a pair of the quotient variety $\mathcal{W}^{|L|,\nu}/\mathrm{Aut}(L)$ and some divisor.

Theorem 1.5 ([20]). *With the notation as in Conjecture 1.4, we have*

$$\sharp_{\mathrm{st}}(\mathcal{Y}, D)_{\overline{C}} = \sum_{L \in G\text{-}\mathrm{Cov}(K)} \frac{\sharp_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L)_{C_L}}{\sharp \mathrm{Aut}(L)}.$$

It is convenient to have a slightly different expression of the right hand side. Let $S_{K,G}$ be the set of continuous homomorphisms $\mathrm{Gal}(K^{\mathrm{sep}}/K) \to G$ from the absolute Galois group of K to G. For an arbitrary function $f : S_{K,G} \to \mathbb{R}_{\geq} \cup \{\infty\}$, we define

$$m(K, G, f) := \frac{1}{\sharp G} \sum_{\rho \in S_{K,G}} f(\rho)$$

and for an arbitrary function $c : S_{K,G} \to \mathbb{R}$, we define

$$M(K, G, c) := \frac{1}{\sharp G} \sum_{\rho \in S_{K,G}} q^{-c(\rho)}.$$

They are related by

$$M(K, G, c) = m(K, G, q^{-c}) \text{ and } m(K, G, f) = M(K, G, -\log_q f).$$

There is a natural map $S_{K,G} \to G\text{-}\mathrm{Cov}(K)$, which is surjective. The fiber over L has cardinality $\sharp G/\sharp \mathrm{Aut}(L)$.

Corollary 1.6. *With the same assumption as above, we have*

$$\sharp_{\mathrm{st}}(\mathcal{Y}, D)_{\overline{C}} = m(K, G, f),$$

where $f = \sharp_{\mathrm{st}}(\mathcal{W}^{|\rho|,\nu}, E_\rho)_{C_\rho}$, changing the super and subscripts from L to ρ in the obvious manner.

If $\mathcal{W}$ is an affine space $\mathbb{A}^d_{\mathcal{O}_K}$ with a linear G-action and $C = \mathcal{W}_k$, then

$$\sharp_{\mathrm{st}}(\mathcal{W}^{|\rho|,\nu}, E_\rho)_{C_\rho} = \sharp_{\mathrm{st}}(\mathbb{A}^d_{\mathcal{O}_K}, -\mathbf{v}(\rho) \cdot \mathbb{A}^d_k) = q^{d - \mathbf{v}(\rho)}.$$

Therefore the equality of the corollary means

$$\sharp_{\mathrm{st}}(\mathcal{Y}, D) = m(K, G, q^{d-\mathbf{v}(\rho)}) = M(K, G, \mathbf{v}) \cdot q^d. \tag{1.3}$$

If we instead put $C := \{o\}$ with $o \in \mathcal{W}_k = \mathbb{A}^d_k$ the origin, then we see that

$$\sharp_{\mathrm{st}}(\mathcal{W}^{|\rho|,\nu}, E_\rho)_{C_\rho} = q^{\mathbf{w}(\rho)},$$

defining $\mathbf{w}$ as follows: let F be the fiber of $\mathcal{W}^{\langle \rho \rangle} \to \mathcal{W}$ over o and

$$\mathbf{w}(\rho) := \dim F - \mathbf{v}(\rho).$$

Thus we get

$$\sharp_{\mathrm{st}}(\mathcal{Y}, D)_{\{\overline{o}\}} = M(K, G, -\mathbf{w}). \tag{1.4}$$

This is basically what was conjectured in [16], with $\mathbf{w}$ slightly modified later. Two formulas (1.3) and (1.4) are closely related by dualities as discussed in [15]. See Section 12.

2. Convention

In what follows, we often consider problems in the absolute setting if possible. Usually the same problems can be asked in the relative setting as well. We often restrict ourselves to linear actions, even if the problem makes sense in the non-linear case too.

We denote by k a perfect field and K a complete discrete valuation field with residue field k. We denote by $\mathcal{O}_K$ the valuation ring of K. We suppose that a finite group action on a variety is always faithful. We denote by p the characteristic of k. When k is finite, we denote its cardinality by q.

We denote by V an affine space $\mathbb{A}^d_k$ over k and by $\mathcal{V}$ an affine space over $\mathcal{O}_K$. By X and $\mathcal{X}$, we denote their quotients by some finite group action respectively. We denote by (X, D) and $(\mathcal{X}, D)$ the log pairs such that the quotient maps $V \to (X, D)$ and $\mathcal{V} \to (\mathcal{X}, D)$ are crepant.

3. Log terminal singularities and convergence

Problem 3.1 ([21])**.** When is the quotient variety $X = \mathbb{A}^d_k/G$ (resp. the log pair (X, D)) log terminal (resp. Kawamata log terminal)?

Log terminal singularities are an important class of singularities in the birational geometry. It is well-known that quotient singularities in characteristic zero are log terminal and that the quotient log pair (X, D) is always Kawamata log terminal. This is no longer true in positive characteristic. Note that X or (X, D) is, by definition, (Kawamata) log terminal if its discrepancy at every divisor over X is > -1. The definition is valid in an arbitrary characteristic. Log terminal singularities are related to the convergence/divergence of stringy invariants and hence the ones of total masses. Indeed, if there exists a log resolution of X, then the following are equivalent:

(1) (X, D) is Kawamata log terminal,
(2) $\sharp_{\mathrm{st}}(X, D) = M(G, K', \mathbf{v}) < \infty$ for all unramified extension K'/K.

Furthermore, these conditions would be equivalent to that $\int_{G\text{-Cov}(K)} \mathbb{L}^{-\mathbf{v}} \neq \infty$, once we could properly define this integral. Without assuming a log resolution, the second condition implies the first. Following , we call (X, D) *stringily log terminal* if the second condition holds.

When G is the cyclic group of prime order $p = \mathrm{char}(k)$, then a simple representation-theoretic characterization was found in [24]. The G-representation $\mathbb{A}^d_k$, identified with the vector space k^d, decomposes into indecomposable representations, say of dimensions $d_1, \dots, d_r$ with $1 \leq d_i \leq p$. We define

$$\mathfrak{D}_G := \sum_{i=1}^{r} \frac{(d_i - 1)d_i}{2}.$$

Remark 3.2. This invariant was denoted as D_V in [24]. We use $\mathfrak{D}$ to distinguish it from divisors.

Then (X, D) is stringily log terminal if and only if $\mathfrak{D}_G \geq p$. Note that if $p \geq 2$, then G has no pseudo-reflection and $D = 0$.

Problem 3.3. Can we generalize this invariant $\mathfrak{D}_G$ for other groups?

Once we could solve these problems, we may consider the following refined problem.

Problem 3.4. What is the minimal discrepancy of (X, D)?

When (X, D) is not stringily log terminal, then $\sharp_{\mathrm{st}}(X, D) = M(K, G, \mathbf{v})q^d = \infty$ at least after a finite unramified base change. The invariants then contain much less information than the other case.

Problem 3.5 ([21])**.** Can we assign finite values to $\sharp_{\mathrm{st}}(X, D)$ and $M(K, G, \mathbf{v})$ even in such a case by a certain "renormalization"?

Veys [14, 13] made such an attempt for stringy invariants in characteristic zero.

4. Rationality and admissibility

Problem 4.1. Suppose that k is algebraically closed. Suppose that Conjecture 1.1 holds. Is $M_{\mathrm{st}}(X, D) = \int_{G\text{-}\mathrm{Cov}(k((t)))} \mathbb{L}^{d-\mathbf{v}}$ a rational function in $\mathbb{L}^{1/\sharp G}$ whenever it is not the infinity?

In the tame case, the answer is positive, because the integral is a finite sum of powers of $\mathbb{L}^{1/\sharp G}$. In the wild case, there seems, a priori, no reason that it is a rational function. However, in all examples I know, the integral turns out to be a rational function. Note that if k is not algebraically closed, the motif is not a rational function in $\mathbb{L}$ even in characteristic zero.

The rationality is closely related to the admissibility in the sense of [15]. Suppose now that k is finite and denote the degree r unramified extension of k by k_r. Consider the function in the variable $r \in \mathbb{Z}_{>0}$,

$$r \mapsto \sharp_{\mathrm{st}}(X \otimes_k k_r) = M(k_r((t)), G, d - \mathbf{v}).$$

Problem 4.2 ([15])**.** Is this function admissible, that is, is it of the form

$$\frac{\sum_{i=1}^{l} n_i \cdot \alpha_i^r}{q^{cr} - 1},$$

where $n_i \in \mathbb{Z}$, $\alpha_i \in \mathbb{C}$ and $c \in \mathbb{Q}^\times$.

If X admits a log resolution, then the admissibility is a consequence of an explicit formula for $\sharp_{\mathrm{st}}$. This means that if the function was not admissible, then X would not admit any log resolution and hence we would get a counterexample for the Hironaka theorem in positive characteristic. Since the absolute Galois group of a local field is well understood, it may be possible to prove the admissibility of $M(k_r((t)), G, d - \mathbf{v})$ without referring to a resolution. On the other hand, modular representations (linear representations of G in the wild case) are very complicated. For instance, it is known that the irreducible representations of a fixed group G are not parametrized by a finite dimensional space. This fact contrasts that there are only finitely many irreducible representations in the tame case. It would not be so surprising if a pathological example was found among them.

5. The function $\mathbf{v}$ and ramification filtration

For $L \in G\text{-}\mathrm{Cov}(K)$, let H be the stabilizer of a connected component of $\operatorname{Spec} L$. It is determined up to conjugacy in G. This group has a natural descending filtration called the *ramification filtration,*

$$H = H_{-1} \supset H_0 \supset H_1 \supset \cdots$$

with $H_i = 1$ for $i \gg 0$.

Problem 5.1. Fix $G \subset \mathrm{GL}_d(k)$. Is the value $\mathbf{v}(L)$ determined by the conjugacy class of H and the ramification filtration?

If $G = \mathbb{Z}/p\mathbb{Z}$, then the ramification filtration is determined by a single integer j called the ramification jump and $\mathbf{v}$ is explicitly written as a function in j (see [24]).

If G consists of permutation matrices, then $\mathbf{v}(L)$ is equal to half the Artin conductor of the associated representation $\mathrm{Gal}(K^{\mathrm{sep}}/K) \to G \hookrightarrow \mathrm{GL}_d(k)$ (see [16]), which is, in turn, determined by the ramification filtration by definition.

6. Crepant resolutions

6.1. Existence.

Problem 6.1 ([19])**.** When does $X = V/G$ have a crepant resolution?

For a finite subgroup $G \subset \mathrm{SL}_d(\mathbb{C})$ with $d = 2$ or 3, there exists a crepant resolution of $\mathbb{C}^d/G$. In dimension two, the minimal resolution is a crepant resolution. In dimension three, this was proved by a case-by-case analysis by Roan, Ito and Markushevich (see [11] and references therein).

In the wild case, only a handful of examples are known. We note that every wild finite subgroup G of $\mathrm{GL}_2(k)$ always has pseudo-reflections. If one does not want pseudo-reflections, the simplest case is dimension three.

When k has characteristic $p > 0$ and G is the cyclic group of order p, then we have the invariant $\mathfrak{D}_G$ mentioned in Section 3. There is no peudo-reflection iff $\mathfrak{D}_G \geq 2$. It was proved in [24] that if $\mathfrak{D}_G \geq 2$ and a crepant resolution of X exists, then $\mathfrak{D}_G = p$. For instance, for $p \geq 3$, suppose that $G \subset \mathrm{SL}_3(k)$ is generated by

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then $\mathfrak{D}_G = 3$. Therefore, for $p > 3$, a crepant resolution does not exist. For $p = 3$, we can construct a crepant resolution. Thus the result of Roan, Ito and Markushevich does not hold in positive characteristic. However, the characteristic three might be exceptional and it might be interesting to ask:

Problem 6.2. Suppose that k has characteristic three. For every small (no pseudo-reflection) finite subgroup of $\mathrm{SL}_3(k)$, is there a crepant resolution of X?

It is also interesting to specialize the problem to

Problem 6.3 ([24])**.** Suppose that $G = \mathbb{Z}/p\mathbb{Z}$ and $\mathfrak{D}_G = p$. In which case is there a crepant resolution of X?

We know the answer only in the following two cases. The first case is that $p = 2$ and G is generated by

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{\oplus 2}$$

and the second case is that $p = 3$ and G is generated by

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

In these cases, there is a crepant resolution of X. Indeed X is then a hypersurface and an easy computation of blowup shows this. For every other case, we do not know whether there is a crepant resolution. Then X is not Cohen-Macaulay, in particular, not a local complete intersection. Therefore it would be difficult to compute blowups explicitly. The first case we should tackle might be the following one:

Problem 6.4 ([24])**.** For an arbitrary prime number p, consider the group $G \subset \mathrm{SL}_{2p}(k)$ generated by

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{\oplus p}.$$

Does there exist a crepant resolution of X?

This singularity seems (at least for me) the wild counterpart of the cyclic quotient singularity in characteristic zero of type $\frac{1}{p}(1, \dots, 1)$ of dimension p. The latter singularity admits a crepant resolution.

As for other examples of crepant resolution, we mention two cases. Firstly let $G = S_n$ act on $\mathbb{A}^{2n}_{\mathcal{O}_K}$ by two copies of the standard permutation representation, the quotient $\mathbb{A}^{2n}_{\mathcal{O}_K}/S_n$ is identical to the symmetric product $S^n\mathbb{A}^2_{\mathcal{O}_K}$ of the affine plane over $\mathcal{O}_K$. It admits a special crepant resolution, namely the Hilbert scheme $\mathrm{Hilb}^n(\mathbb{A}^2_{\mathcal{O}_K})$ of n points. This is wild if $n \ge p$ and provides an infinite series of examples of crepant resolutions.

The other case is similar. Suppose that $\mathrm{char}(k) \ne 2$. Let $G \subset \mathrm{GL}_d(\mathcal{O}_K)$ be the group of signed permutation matrices and consider its diagonal action on $\mathbb{A}^{2n}_{\mathcal{O}_K} = \mathbb{A}^n_{\mathcal{O}_K} \times_{\mathrm{Spec}\,\mathcal{O}_K} \mathbb{A}^n_{\mathcal{O}_K}$. We can construct a crepant resolution of $\mathbb{A}^{2n}_{\mathcal{O}_K}/G$ by using the Hilbert scheme of points. See [15] for details.

Problem 6.5. Find more examples of crepant resolutions constructed as moduli spaces.

The ones mentioned above are all I know as examples of crepant resolutions in the wild case, excepting the non-linear ones. Any new example is very welcome.

6.2. **Euler characteristics.**

Problem 6.6. Let $G \subset \mathrm{GL}_d(k)$ be a small subgroup and $Y \to X = V/G$ a crepant resolution. Is the Euler characteristic of $Y_{\bar{k}}$ (with respect to l-adic étale cohomology) equal to $\sharp\mathrm{Conj}(G)$?

In the tame case, this is a direct consequence of (1.1). Roughly speaking, the Euler characteristic of Y is obtained by substituting 1 for $\mathbb{L}$ and the right hand side of (1.1) becomes $\sharp\mathrm{Conj}(G)$ by this substitution. In fact, this shows that the *stringy* Euler characteristic of X is always equal to $\sharp\mathrm{Conj}(G)$ in the tame case, without assuming a crepant resolution. These are no longer true in the wild case. For instance, the stringy Euler characteristic is not generally an integer but only a rational number (see [24] for an example). There seems no reason that the answer to the problem is positive. However, it is actually positive for $G = \mathbb{Z}/p\mathbb{Z}$ and also for all computed examples so far.

7. Log resolutions

Problem 7.1. Compute explicitly log resolution of singularities for some class of wild quotient singularities?

Desired are not only crepant resolutions but also log resolutions in general. Once we could computed a log resolution of X explicitly, then we would have an explicit formula for $M_{\mathrm{st}}(X)$ and $\sharp_{\mathrm{st}}(X)$, and for $\int_{G\text{-Cov}(K)} \mathbb{L}^{d-\mathbf{v}}$ and $M(K, G, \mathbf{v})$; the latters are quantities interesting from the number-theoretic perspective.

8. Untwisting

8.1. **Singularities of untwisting varieties.** Let $\mathcal{W}$ be an affine $\mathcal{O}_K$-variety with a G-action and $\mathcal{W} \subset \mathcal{V}$ an equivariant embedding into an affine space $\mathcal{V} = \mathbb{A}^d_{\mathcal{O}_K}$ as in section 1.3. Even if $\mathcal{W}$ is regular or smooth over $\mathcal{O}_K$, the untwisting varieties $\mathcal{W}^{|L|}$ and their normalizations $\mathcal{W}^{|L|,\nu}$ are not generally so. A general question we would like to ask is:

Problem 8.1 ([21])**.** What kind of singularities do varieties $\mathcal{W}^{|L|}$, $\mathcal{W}^{|L|,\nu}$ and the pair $(\mathcal{W}^{|L|,\nu}, E_L)$ have?

As more specific problems, we firstly consider the following:

Problem 8.2 ([21])**.** If $\mathcal{W}$ is normal, then are $\mathcal{W}^{|L|}$ normal?

If this is the case, then we do not need to take the normalization $\mathcal{W}^{|L|,\nu}$.

If $\mathcal{W}$ is a hypersurface in $\mathcal{V}$, then $\mathcal{W}^{|L|}$ is also so in $\mathcal{V}^{|L|}$ by the reason of dimension.

Problem 8.3 ([21])**.** If $\mathcal{W} \subset \mathcal{V}$ is a complete intersection, then are $\mathcal{W}^{|L|} \subset \mathcal{V}^{|L|}$ so?

Problem 8.4. If $(\mathcal{W}, E)$ is stringily log terminal, then are $(\mathcal{W}^{|L|,\nu}, E_L)$ so?

If this is true, then in the sum of Theorem 1.5, at least each term $\frac{\sharp_{\mathrm{st}}(\mathcal{W}^{|L|,\nu}, E_L)_{C_L}}{\sharp \mathrm{Aut}(L)}$ is finite.

In one example computed in [21], k has characteristic two and $\mathcal{W} \subset \mathcal{V} = \mathbb{A}^2_{k[[t]]}$ is a hypersurface stable under the transposition of coordinates. In this example, the untwisting varieties $W^{|L|}$, whose total dimension is two, have some rational double points, which one can find in Artin's classification [1].

Problem 8.5. Determine singularities appearing on $\mathcal{W}^{|L|}$ for more examples where $\mathcal{W}$ is a hypersurface in $\mathcal{V} = \mathbb{A}^2_{\mathcal{O}_K}$, and compute stringy invariants of $(\mathcal{W}^{|L|,\nu}, E_L)$ explicitly.

8.2. **An intrinsic untwisting.**

Problem 8.6. Can we construct untwisting varieties $\mathcal{W}^{|L|}$ or their alternatives in an intrinsic manner?

The untwisting technique, as presented in section 1.3, is extrinsic by nature: we need to embed the given affine variety $\mathcal{W}$ into an affine space. This makes it difficult to generalize the wild McKay correspondence to the non-affine case. Therefore an intrinsic construction is desired.

9. The moduli space G-Cov(K)

Problem 9.1 ([19])**.** Construct the moduli space (stack) G-Cov(K) and describe its geometry.

When $K = k((t))$ with k algebraically closed and G is a p-group, Harbater [6] showed that the coarse moduli space is the direct limit of affine spaces $\mathbb{A}_k^n$ with respect to the composite morphisms $\mathbb{A}_k^n \to \mathbb{A}_k^{n+1}$ of the standard inclusions and the Frobenius morphisms.

We would like to have fine moduli stacks for general K, which would be necessary to have the motivic integration over Deligne-Mumford stacks in full generality (see Section 10).

Problem 9.2. Show that every fiber of $\mathbf{v} : G\text{-Cov}(K) \to \mathbb{Q}$ is a finite dimensional constructible set.

If values of $\mathbf{v}$ are determined by ramification filtration (see Section 5), then the problem reduced to the following problem.

Problem 9.3. Fix a subgroup $H \subset G$ and a filtration $H \supset H_0 \supset H_1 \supset \cdots$. The moduli space of G-covers of K with this ramification filtration is a finite-dimensional k-variety or at least a constructible subset.

10. Deligne-Mumford stacks and finite group schemes

Problem 10.1 ([19])**.** Generalize the motivic integration theory to Deligne-Mumford stacks over $\mathcal{O}_K$ and prove Conjecture 1.4.

This would be a natural strategy to prove Conjecture 1.4. This would also prove a generalization of them: for a proper birational crepant morphism $f : (\mathcal{Y}, E) \to (\mathcal{X}, D)$ of log Deligne-Mumford stacks over $\mathcal{O}_K$ (pairs of stacks and $\mathbb{Q}$-divisors) and for a constructible subset of $\mathcal{X}_k$, we have the equality of suitably defined stringy invariants of the pairs,

$$M_{\mathrm{st}}(\mathcal{X}, D)_C = M_{\mathrm{st}}(\mathcal{Y}, E)_{f^{-1}C}. \tag{10.1}$$

Moreover this invariant should be defined in such a way that if $\mathcal{X}$ is the quotient stack $[\mathcal{V}/G]$, then $M_{\mathrm{st}}(\mathcal{X})$ is easily expressed as $\int_{G\text{-Cov}(G)} \mathbb{L}^{d-\mathbf{v}}$.

In the tame case, generalization as in the problem has been accomplished to a considerable extent in [23], after special cases had been studied in [22, 10].

For this problem problem, the first thing to do would be construction of an appropriate space of arcs and jets.

Problem 10.2 ([19])**.** Construct the moduli stacks of twisted arcs/jets as did in [23] in the tame case.

Roughly, for a Deligne-Mumford stack $\mathcal{X}$ over $\mathcal{O}_K$, a twisted arc is a representable morphism of the form

$$[\operatorname{Spec} \mathcal{O}_L/G] \to \mathcal{X},$$

where G is a finite group and $\operatorname{Spec} L \to \operatorname{Spec} K$ is a G-cover. The stack should have the functoriality: if $\mathcal{J}_\infty\mathcal{X}$ denotes the moduli stack of twisted arcs of $\mathcal{X}$ and if $\mathcal{Y} \to \mathcal{X}$ is a morphism of Deligne-Mumford stacks, then we would have a morphism $\mathcal{J}_\infty\mathcal{Y} \to \mathcal{J}_\infty\mathcal{X}$.

The invariant $M_{\mathrm{st}}(\mathcal{X}, D)_C$ should be expressed as

$$\int_{(\mathcal{J}_\infty\mathcal{X})_C} \mathbb{L}^{F_{\mathcal{X},D}},$$

where $(\mathcal{J}_\infty\mathcal{X})_C$ is the moduli stack of twisted arcs sending the closed point into C with an appropriate motivic measure and $F_{\mathcal{X},D}$ is a function on it canonically determined by the pair $(\mathcal{X}, D)$. Equality (10.1) would become a direct consequence of the change of variables formula, once it was proved as a part of the theory.

11. The derived wild McKay correspondence

Problem 11.1 ([24])**.** Generalize the McKay correspondence at the level of derived categories as studied in [9, 3] to the wild case.

Suppose that there exists a crepant resolution $Y \to X = \mathbb{A}_k^d/G$. The derived McKay correspondence means the equivalence

$$D^b(\mathrm{Coh}(Y)) \cong D^b(\mathrm{Coh}^G(\mathbb{A}_k^d)),$$

between the bounded derived category of coherent sheaves of Y and the one of equivariant coherent sheaves of $\mathbb{A}_k^d$. This was proved under some conditions in characteristic zero.

However, the category $\mathrm{Coh}^G(\mathbb{A}_k^d)$ always has infinite global dimension in the wild case [25], while it has global dimension d in the tame case. Therefore, the equivalence above never holds in the wild case. We need to find an alternative to $\mathrm{Coh}^G(\mathbb{A}_k^d)$.

12. Dualities and equisingularities

Suppose that k is finite. For $G \subset \mathrm{GL}_d(\mathcal{O}_K)$, considering all unramified extensions K_r, $r \geq 1$ of K, we get two functions $M(K_r, G, \mathbf{v})$ and $M(K_r, G, -\mathbf{w})$ in the variable r. Assuming they are admissible functions, we can define their duals $\mathbb{D}(-)$ in such a way that the dual of the function q^r is q^{-r}. We consider the following two equalities:

$$\mathbb{D}(M(K_r, G, \mathbf{v})) = M(K_r, G, -\mathbf{w}), \tag{12.1}$$

$$M(K_r, G, \mathbf{v})q^{rd} - M(K_r, G, -\mathbf{w}) = \mathbb{D}(M(K_r, G, -\mathbf{w}))q^{rd} - M(K_r, G, \mathbf{v}). \tag{12.2}$$

When the former (resp. the latter) holds, we say that the *strong (resp. weak) duality* holds. The strong duality implies the weak duality.

Problem 12.1 ([15])**.** If $K = k((t))$ and the G-action on $\mathbb{A}_{k[[t]]}^d$ is defined over k (that is, it is the base change of a G-action on $\mathbb{A}_k^d$), then does the strong duality always holds? What about the weak duality?

In all the examples computed at the present, the strong duality as well as the weak duality holds. If $\mathbb{A}_k^d/G$ has a $\mathbb{G}_m$-equivariant log resolution, then the weak duality holds as a consequence of the Poincaré duality for $\sharp_{\mathrm{st}}$. On the contrary, if one found an example without satisfying the weak duality, the corresponding k-variety $\mathbb{A}_k^d/G$ does not have any $\mathbb{G}_m$-equivariant log resolution.

Problem 12.2 ([15])**.** Suppose that G acts on $\mathbb{A}_{\mathcal{O}_K}^d$ by permutation of coordinates. Does the weak duality always hold? Does $X = \mathbb{A}_{\mathcal{O}_K}^d/G$ always admit a simultaneous resolution over $\mathcal{O}_K$?

The problem is interesting when $\mathcal{O}_K$ has mixed characteristic. The second problem may be expressed as: is X an equisingular family over $\mathcal{O}_K$? In computed examples, the weak duality always holds but the strong duality does not.

References

[1] M. Artin. Coverings of the rational double points in characteristic p. In *Complex analysis and algebraic geometry*, pages 11–22. Iwanami Shoten, Tokyo, 1977.

[2] Victor V. Batyrev. Non-Archimedean integrals and stringy Euler numbers of log-terminal pairs. *J. Eur. Math. Soc. (JEMS)*, 1(1):5–33, 1999.

[3] Tom Bridgeland, Alastair King, and Miles Reid. The McKay correspondence as an equivalence of derived categories. *J. Amer. Math. Soc.*, 14(3):535–554 (electronic), 2001.

[4] Jan Denef and François Loeser. Motivic integration, quotient singularities and the McKay correspondence. *Compositio Math.*, 131(3):267–290, 2002.

[5] Christopher D. Hacon and James McKernan. Existence of minimal models for varieties of log general type. II. *J. Amer. Math. Soc.*, 23(2):469–490, 2010.

[6] David Harbater. Moduli of p-covers of curves. *Comm. Algebra*, 8(12):1095–1122, 1980.

[7] Hiroyuki Ito and Stefan Schroeer. Wild quotient surface singularities whose dual graphs are not star-shaped. arXiv:1209.3605, 09 2012.

[8] Yukari Ito and Miles Reid. The McKay correspondence for finite subgroups of $\mathrm{SL}(3, \mathbf{C})$. In *Higher-dimensional complex varieties (Trento, 1994)*, pages 221–240. de Gruyter, Berlin, 1996.

[9] M. Kapranov and E. Vasserot. Kleinian singularities, derived categories and Hall algebras. *Math. Ann.*, 316(3):565–576, 2000.

[10] Ernesto Lupercio and Mainak Poddar. The global McKay-Ruan correspondence via motivic integration. *Bull. London Math. Soc.*, 36(4):509–515, 2004.

[11] Shi-Shyr Roan. Minimal resolutions of Gorenstein orbifolds in dimension three. *Topology*, 35(2):489–508, 1996.

[12] Julien Sebag. Intégration motivique sur les schémas formels. *Bull. Soc. Math. France*, 132(1):1–54, 2004.

[13] Willem Veys. Stringy zeta functions for $\mathbb{Q}$-Gorenstein varieties. *Duke Math. J.*, 120(3):469–514, 2003.

[14] Willem Veys. Stringy invariants of normal surfaces. *J. Algebraic Geom.*, 13(1):115–141, 2004.

[15] Melanie Matchett Wood and Takehiko Yasuda. Mass formulas for local Galois representations and quotient singularities II: dualities and resolution of singularities. arXiv:1505.07577.

[16] Melanie Matchett Wood and Takehiko Yasuda. Mass formulas for local Galois representations and quotient singularities I: a comparison of counting functions. *International Mathematics Research Notices*, doi: 10.1093/imrn/rnv074, 2015.

[17] Takehiko Yasuda. Densities of rational points and number fields. arXiv:1408.3912.

[18] Takehiko Yasuda. Manin's conjecture vs. malle's conjecture. arXiv:1505.04555.

[19] Takehiko Yasuda. Toward motivic integration over wild Deligne-Mumford stacks. arXiv:1302.2982, to appear in the proceedings of "Higher Dimensional Algebraic Geometry - in honour of Professor Yujiro Kawamata's sixtieth birthday".

[20] Takehiko Yasuda. The wild McKay correspondence and p-adic measures. arXiv:1412.5260.

[21] Takehiko Yasuda. Wilder McKay correspondences. arXiv:1404.3373, to appear in Nagoya Mathematical Journal.

[22] Takehiko Yasuda. Twisted jets, motivic measures and orbifold cohomology. *Compos. Math.*, 140(2):396–422, 2004.

[23] Takehiko Yasuda. Motivic integration over Deligne-Mumford stacks. *Adv. Math.*, 207(2):707–761, 2006.

[24] Takehiko Yasuda. The p-cyclic McKay correspondence via motivic integration. *Compos. Math.*, 150(7):1125–1168, 2014.

[25] Zhong Yi. Homological dimension of skew group rings and crossed products. *J. Algebra*, 164(1):101–123, 1994.

Department of Mathematics, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan, tel:+81-6-6850-5326, fax:+81-6-6850-5327

E-mail address: takehikoyasuda@math.sci.osaka-u.ac.jp, highernash@gmail.com