Introduction to Euler Getter

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Mathematical Software and Free Documents XV

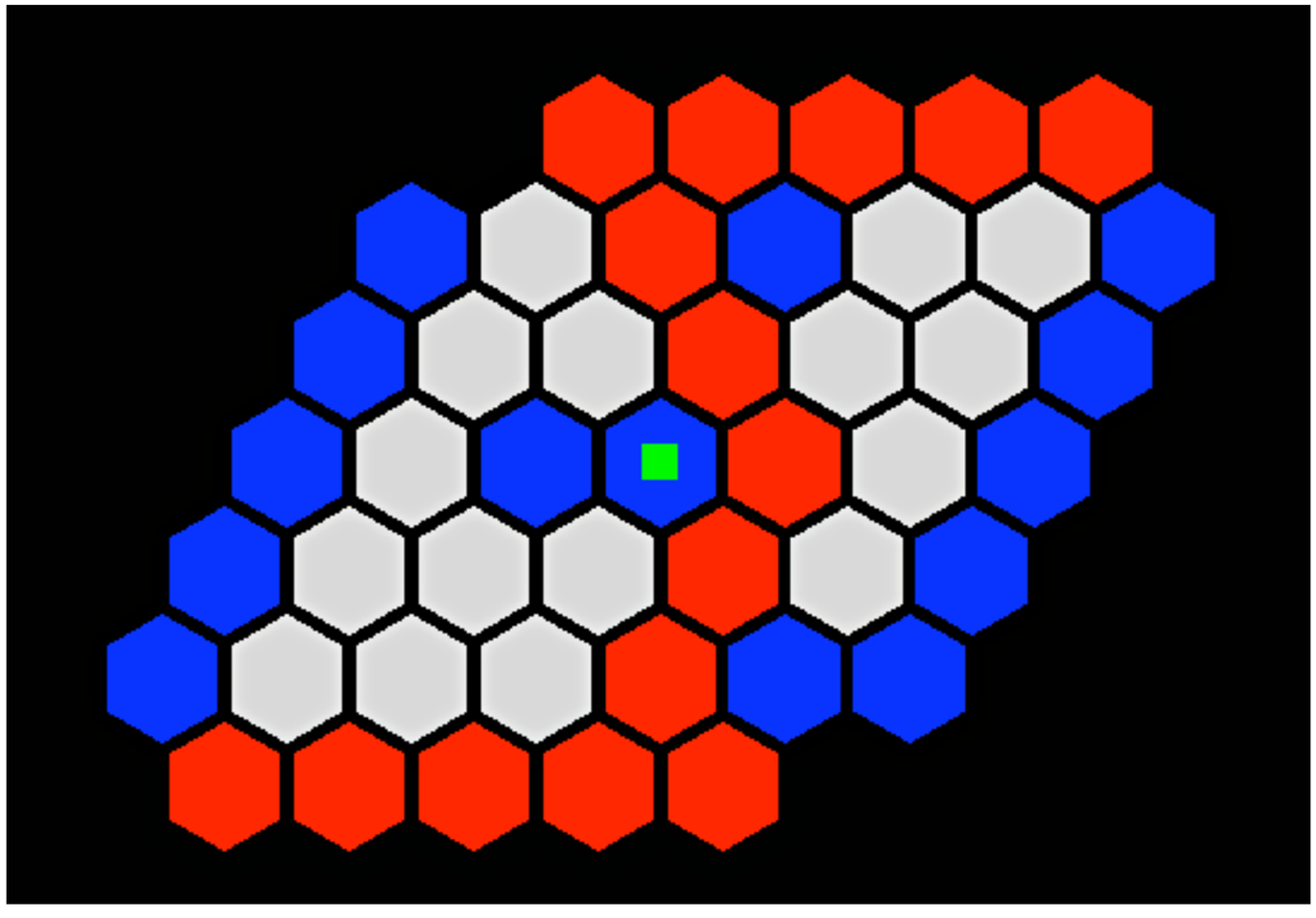
What is EG (Euler Getter) ?

a game introduced by Y. (2010)

Features	Other games with the feature
topological (connection)	Hex, Minor, Shapley
territory	Go, Reversi (Othello)

Hex

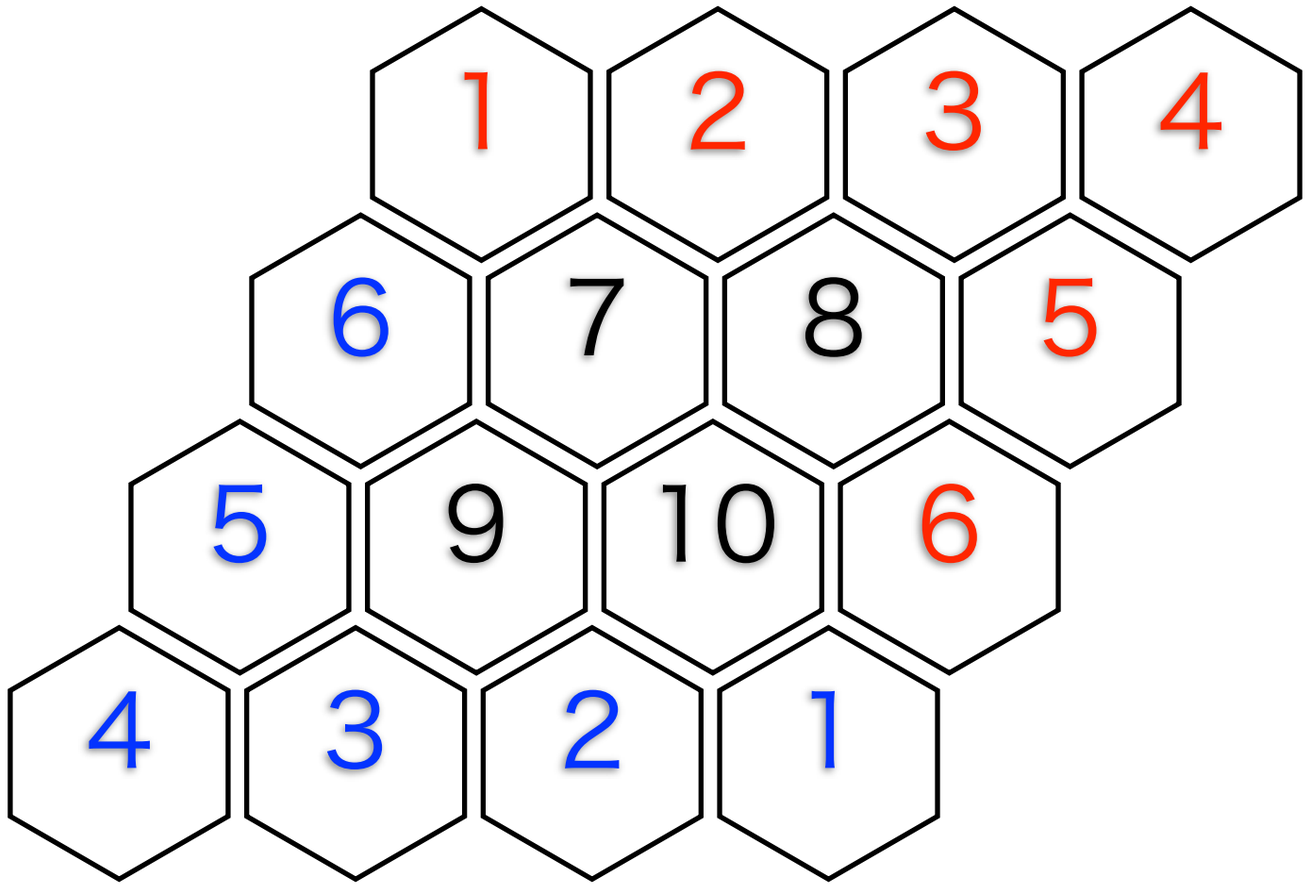
the **first topological game** invented by Piet Hein
and reinvented by John Nash (1940s)



Hex variants

- Milnor (or Y)
- Shapley (or Projective Plane, Projex)
- many others

EG Board



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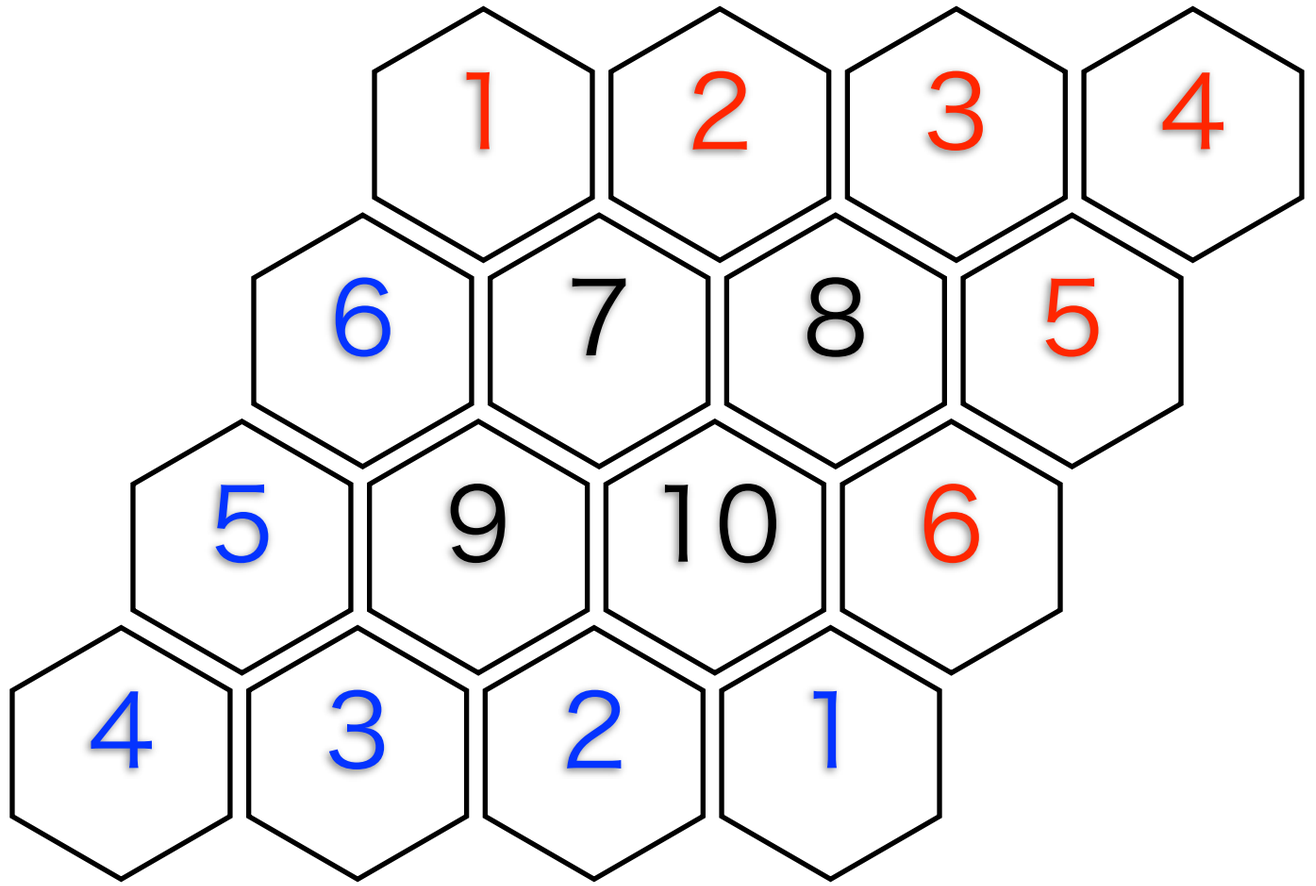

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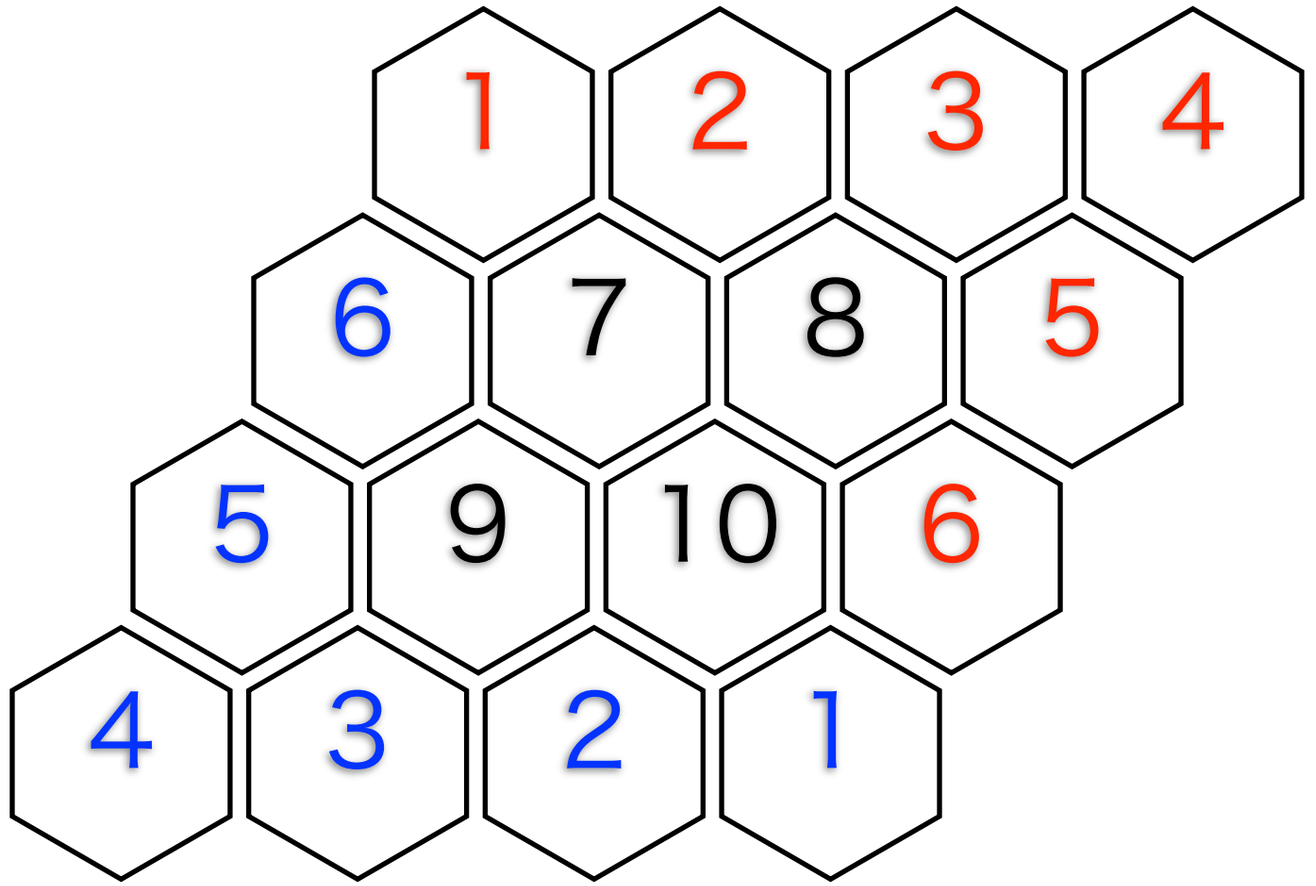

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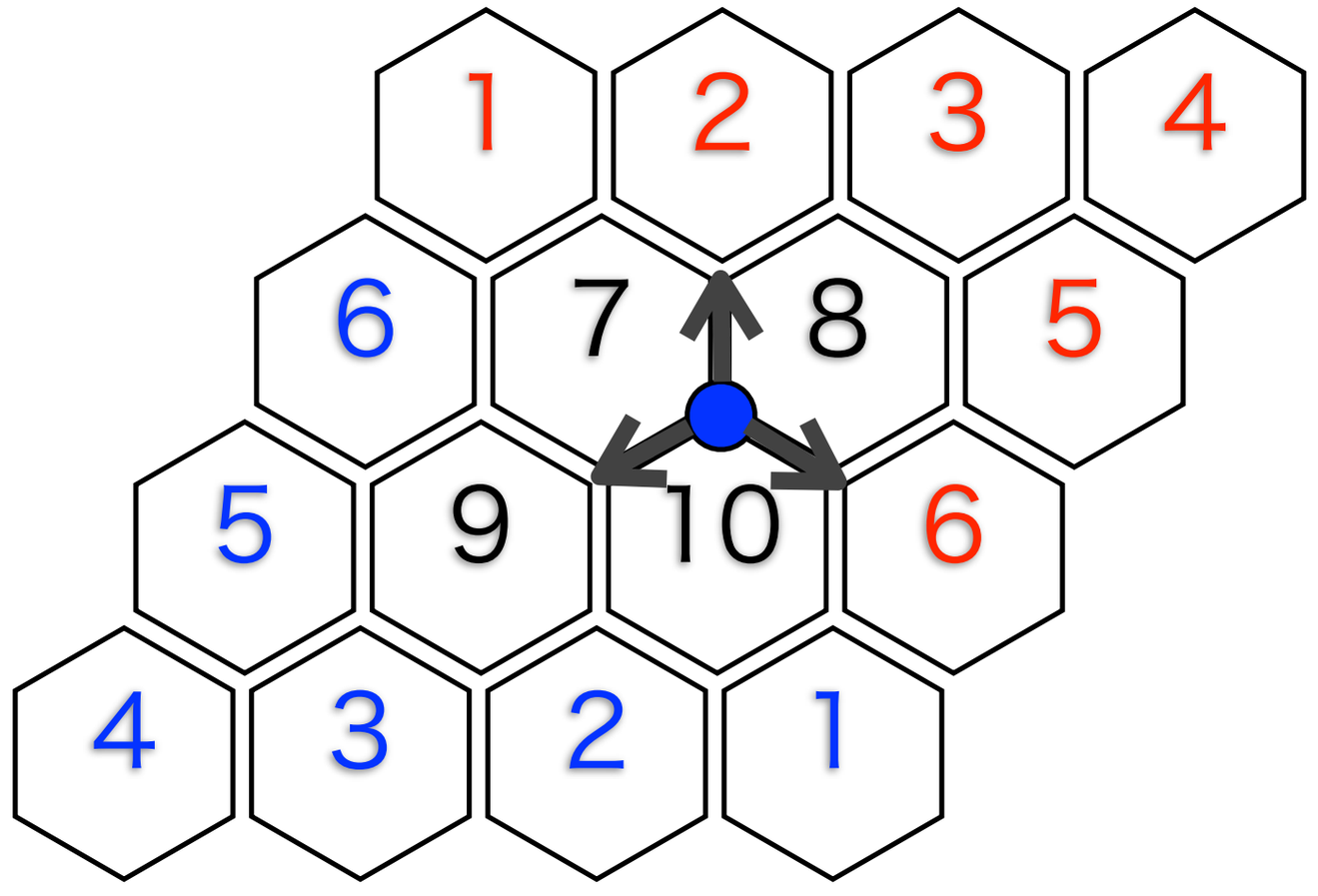

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- The winner is the one whose area has larger **Euler characteristic**.

Euler characteristic

A : area consisting of cells on a EG board

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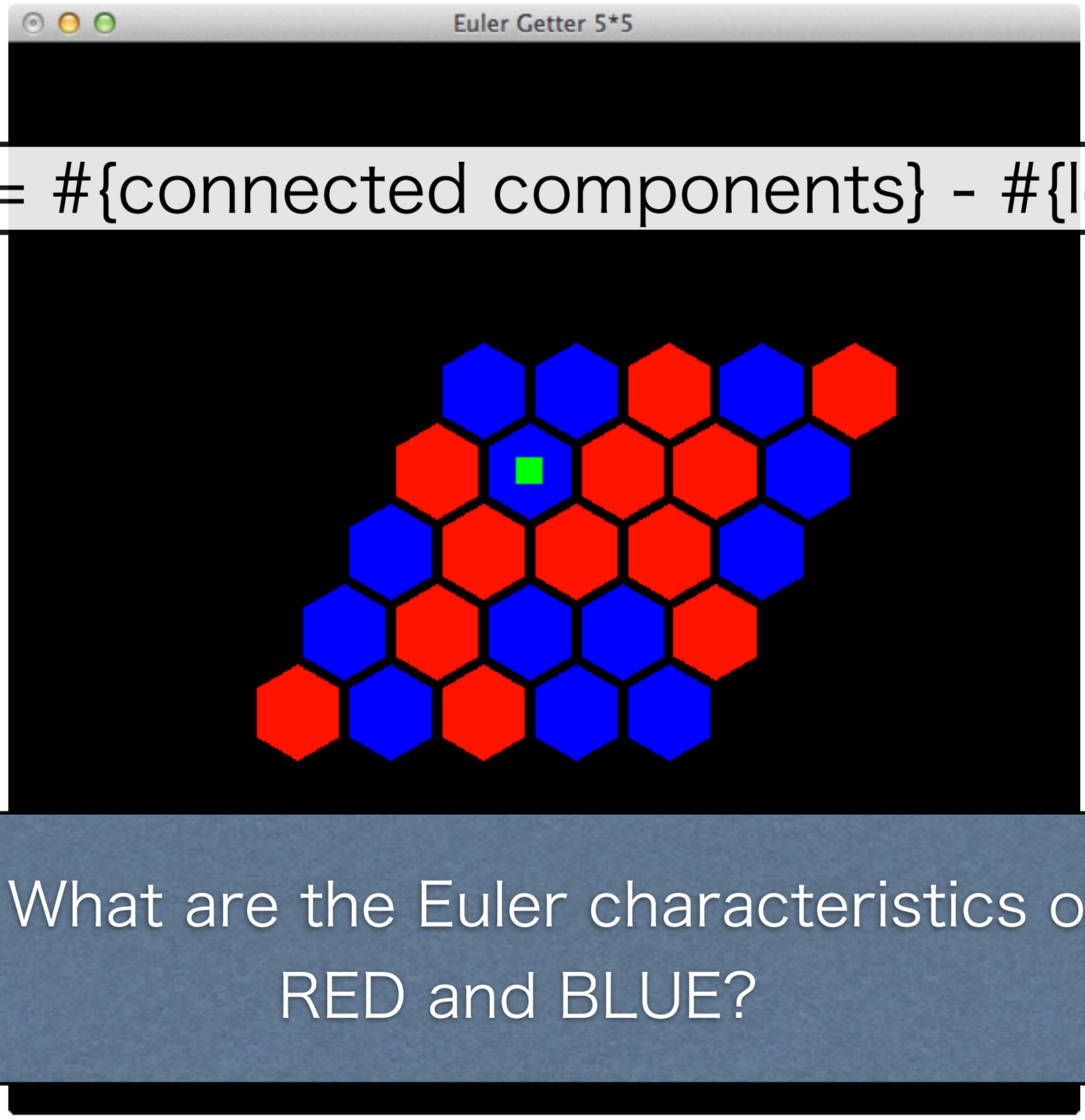
$e(A) := \#\{\text{vertices}\} - \#\{\text{edges}\} + \#\{\text{cells}\}$

$= \#\{\text{connected components}\} - \#\{\text{loops}\}$ ← human-friendly

Example

e(A) = #{connected components} - #{loops}



Q: What are the Euler characteristics of RED and BLUE?

Example

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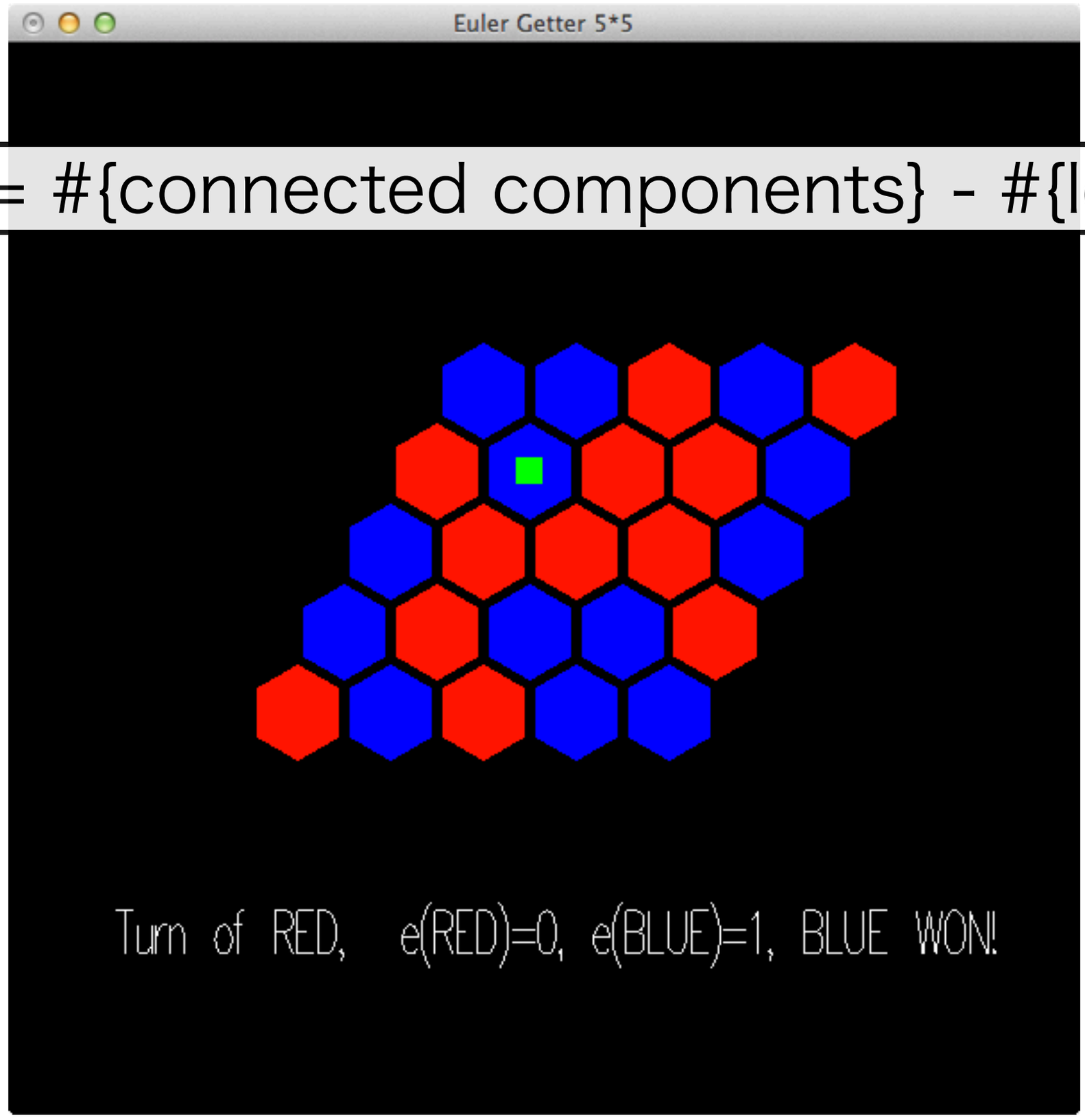

Euler characteristic as a **measure**

Inclusion-exclusion principle:

$$e(X \cup Y) = e(X) + e(Y) - e(X \cap Y)$$

The idea came from the motivic integration.

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Key Facts

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- Red ∩ Blue = disjoint loops
- e(loop) = 0

Winning Strategy

Theorem (Schnell)

If #{cells} is even, then the first player has a winning strategy.

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Proof

Strategy-stealing argument

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- 渋田止め (Shibuta block)

Implementations

In chronological order,

- Euler Getter 1 (Y., Haskell)
- Web Euler Getter (motemen, Perl+JavaScript)
- E2G2 (Hashimoto, Maxima, **AI**)
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The Monte-Carlo method works well.
(Or humans are still too weak.)

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- Difficult to explain rules to the general public
- No iOS or Android implementation

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a possible variant of EG

which might address issues in the last slide

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Algebro-geometric interpretation

torus + scores = log elliptic curve

Euler char. + scores = stringy Euler number

References

- Euler Getter Wiki:

  http://www14.atwiki.jp/euler_getter/

- My homepage:

  http://takehikoyasuda.jimdo.com/