Corrigendum to "Some functional inequalities on non-reversible Finsler manifolds" [Proc. Indian Acad. Sci. Math. Sci. 127 (2017) 833–855]

Shin-ichi Ohta*

Abstract

We correct the proof of the Sobolev-type inequality in [2] for $1 < p < 2$ (called the Beckner inequality).

In [2, Theorem 5.6], we state the following Sobolev-type inequality on a Finsler manifold (M, F) equipped with a measure $\mathfrak{m}$.

Theorem 1 *Assume that* $\mathrm{Ric}_N \geq K > 0$ *for some* $N \in [n, \infty)$ *and* $\mathfrak{m}(M) = 1$*. Then we have*

$$\frac{\|f\|_{L^p}^2 - \|f\|_{L^2}^2}{p-2} \leq \frac{N-1}{KN} \int_M F^2(\boldsymbol{\nabla} f)\,\mathrm{d}\mathfrak{m} \tag{1}$$

for all $1 \leq p \leq 2(N+1)/N$ *and* $f \in H^1(M)$*.*

The proof in [2] is, however, incorrect for $1 < p < 2$ (precisely, the final approximation procedure requires $p > 2$). Instead, we can apply the argument in [1] to show (1) for $1 < p < 2$ (such an inequality is called the *Beckner inequality*). Furthermore, the argument in [1] gives the following generalization of Theorem 1.

Theorem 2 *Assume that* $(M, F, \mathfrak{m})$ *is compact and satisfies* $\mathrm{Ric}_N \geq K > 0$ *for some* $N \in (-\infty, -2)$ *and* $\mathfrak{m}(M) = 1$*. Then we have*

$$\frac{\|f\|_{L^p}^2 - \|f\|_{L^2}^2}{p-2} \leq \frac{N-1}{KN} \int_M F^2(\boldsymbol{\nabla} f)\,\mathrm{d}\mathfrak{m}$$

for all $1 \leq p \leq (2N^2+1)/(N-1)^2$ *and* $f \in H^1(M)$*.*

We refer to a forthcoming book [3] for details and further discussions.

References

[1] I. Gentil and S. Zugmeyer, A family of Beckner inequalities under various curvature-dimension conditions. Bernoulli **27** (2021), 751–771.

[2] S. Ohta, Some functional inequalities on non-reversible Finsler manifolds. Proc. Indian Acad. Sci. Math. Sci. **127** (2017), 833–855.

[3] S. Ohta, Comparison Finsler geometry. Book in preparation.

*Department of Mathematics, Osaka University, Osaka 560-0043, Japan (s.ohta@math.sci.osaka-u.ac.jp)