

Galois-theoretic Arithmetic Geometry

Dates: October 19-24, 2010

Venue: International Institute for Advances Studies and Keihanna Plaza Hotel, Kyoto, Japan

Organizing Committee: H. Nakamura (Chair); F. Pop; L. Schneps; A. Tamagawa; Y. Hoshi

Program

19 Tuesday

Arrival Day

19:00-23:00 Night Session

20 Wednesday

09:40-09:45 H. Nakamura Opening Remarks

09:45-10:45 K. Hashimoto Noether's Problem for transitive permutation groups of degree 6 and the moduli spaces of genus 2 curves with prescribed endomorphism structures of their jacobians

11:00-12:00 M. Kakde *L*-functions and *K*-theory

12:00-13:30 Lunch Break

13:30-14:30 Programme du jour

14:45-15:45 H. Tokunaga Splitting curves in double covers and elliptic surfaces

16:00-17:00 Z. Wojtkowiak Periods of mixed Tate motives - ℓ -adic Galois side, examples

17:00-19:00 Dinner Break

19:00-23:00 Night Session

21 Thursday

09:45-10:45 S. Morita Structure of the space of symplectic invariant tensors and its applications in the tautological algebra of the moduli space of curves and beyond

11:00-12:00 N. Imai Dimensions of moduli spaces of finite flat models

12:00-13:30 Lunch Break

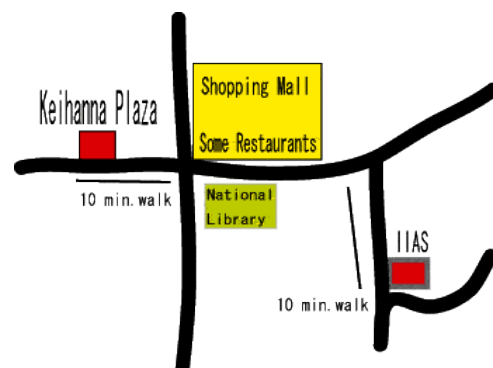
13:30-14:30 Programme du jour

14:45-15:45 C. Rasmussen Finiteness results on abelian varieties with constrained torsion

16:00-17:00 A. Obus Lifting Cyclic Extensions

17:00-19:00 Dinner Break

19:00-23:00 Night Session



22 Friday

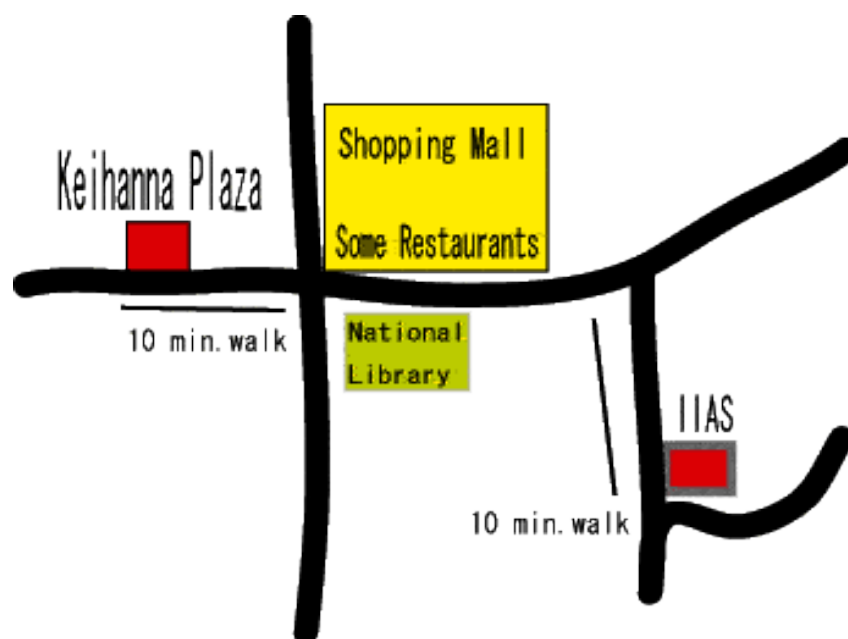
09:45-10:45	S. Carr	Moulds and multiple zeta values
11:00-12:00	S. Corry	Galois branched covers of finite graphs
12:00-13:30	Lunch Break	
13:30-14:30	Programme du jour	
14:45-15:45	T. Ichikawa	Motivic properties of modular groups
16:00-17:00	P. Debes	The Regular Inverse Galois Problem and the Hilbert-Grunwald property
18:30-20:30	Banquet	
20:00-23:00	Night Session	

23 Saturday

09:45-10:45	J. Tong	On the variation of fundamental groups
11:00-12:00	A. Auel	A Milnor conjecture for p -adic curves
12:00-13:30	Lunch Break	
13:30-14:30	Programme du jour	
14:45-15:45	G. Yamashita	p -adic multiple zeta values and motivic Galois group
16:00-17:00	G. Quick	Galois actions on etale homotopy types
17:00-19:00	Dinner Break	
19:00-23:00	Night Session	

24 Sunday

Departure Day
(Rental Bus to Kyoto Hotels)



Abstracts

October 20 Wednesday

09:40-09:45: **H. Nakamura**

Opening Remarks

09:45-10:45: **K. Hashimoto**

Title: Noether's Problem for transitive permutation groups of degree 6 and the moduli spaces of genus 2 curves with prescribed endomorphism structures of their jacobians

Abstract: The function field of the moduli space of ordered six points of the projective line modulo the action of PGL_2 is identified, via cross-ratios of these points, with the rational function field of 3 variables, on which we have a natural action of S_6 (symmetric group of degree 6) induced from the permutation of six points. We will discuss the Noether's problem in this setting for various transitive permutation groups of degree 6 (there are 16 such groups), and the possible connection to the geometry of the moduli space of genus two curves which are obtained as double covers of the projective line ramifying exactly at these six points.

11:00-12:00: **M. Kakde**

Title: L -functions and K -theory

13:30-14:30: Programme du jour

14:45-15:45: **H. Tokunaga**

Title: Splitting curves in double covers and elliptic surfaces

Abstract Let Σ be a smooth projective surface. Let $f : Z \rightarrow \Sigma$ be a double cover of Σ with branch locus B and let Z' be the Stein factorization of f . An irreducible curve on D is called a splitting curve with respect to f if f^*D is of the form

$$f^*D = D^+ + D^- + E,$$

where $D^+ \neq D^-$, $\sigma_f^*D^+ = D^-$, $f(D^+) = f(D^-) = D$ and $\text{Supp}(E)$ is contained in the exceptional set of $\mu : Z \rightarrow Z'$. In this talk, we consider some properties in the case when $\Sigma =$ a rational ruled surface, $Z =$ a double cover of Σ branched at two disjoint sections and $D =$ a tri-sections on Σ .

16:00-17:00: **Z. Wojtkowiak**

Title: Periods of mixed Tate motives - ℓ -adic Galois side, examples

Abstract: Iterated integrals from 0 to 1 in sequences of one forms dz/z and $dz/(z-1)$ are periods of mixed Tate motives over $\text{Spec } \mathbb{Z}$. The obvious question is if in this way we get all such periods. We shall discuss the Galois, ℓ -adic version of this problem over \mathbb{Z} and over any ring of algebraic S -integers.

October 21 Thursday

09:45-10:45: **S. Morita**

Title: Structure of the space of symplectic invariant tensors and its applications in the tautological algebra of the moduli space of curves and beyond

Abstract: We define a natural inner product in the space of symplectic invariant tensors of any given genus and dimension, and give a direct sum decomposition of this space according to its eigenvalues. This gives a complete description how the above space degenerates when we decrease the genus one by one from the stable range to genus one. We then mention several applications of this structure in various problems related to surfaces: tautological algebra of the moduli space of curves related to Faber conjecture, its possible extension to the arithmetic context, cohomology of the moduli space of graphs and finally a series of invariants for certain topological 4-manifolds which we expect to detect the difference between topological versus smooth structures.

11:00-12:00: **N. Imai**

Title: Dimensions of moduli spaces of finite flat models

Abstract: A finite flat model is a finite flat group scheme over the ring of integers of a p -adic field with a fixed generic fiber. In this talk, we discuss a moduli space of finite flat models. Especially, we give a dimensional bound of the moduli spaces using a ramification index of a p -adic field.

13:30-14:30: Programme du jour

14:45-15:45: **C. Rasmussen**

Title: Finiteness results on abelian varieties with constrained torsion

Abstract: We consider a finiteness question in the spirit of the Shafarevich Conjecture. Rather than fixing a specific reduction type for an abelian variety, we let it vary in a controlled fashion. In exchange for this freedom, we place an arithmetic constraint on torsion. Conjecturally, the set of isomorphism classes should be finite, and this has been proven in several cases. We report on the current status of the conjecture. (joint with Akio Tamagawa)

16:00-17:00: **A. Obus**

Title: Lifting Cyclic Extensions

Abstract: The "local lifting problem" asks: given a finite group G and an embedding $i: G \rightarrow$ (continuous k -automorphisms of $k[[t]]$), where k is algebraically closed of characteristic p , does there exist a finite extension $R/W(k)$ and a map $j: G \rightarrow$ (continuous R -automorphisms of $R[[t]]$) such that j lifts i ? The Oort conjecture states that the local lifting problem should always have a solution for G cyclic. This is basic Kummer theory when p does not divide $|G|$, and has been proven when $v_p(|G|) = 1$ (Oort, Sekiguchi, Suwa) and when $v_p(|G|) = 2$ (Green, Matignon). We will first motivate the local lifting problem from geometry, and then we will show that it has a solution for a large family of cyclic extensions. This family includes all extensions where $v_p(|G|) = 3$ and many extensions where $v_p(|G|)$ is arbitrarily high. This is joint work with Stefan Wewers.

October 22 Friday

09:45-10:45: **S. Carr**

Title: Moulds and multiple zeta values

Abstract: The goal of this talk is to outline some of J. Ecalle's main results on multiple zeta values. I will first give an introduction to moulds and multiple zeta values. One of the important ingredients in Ecalle's work on multiple zeta values is his explicit construction, starting with the mould, zag, of formal generalized multiple zeta values, of a canonical set of generators of the multiple zeta value algebra. In the second part of the talk, I will give this construction and then explain how the mould zag can be decomposed into three factors with respect to the Lie group product, gari. This decomposition shows that formal multiple zeta value algebra is freely generated by $\zeta(2)$ and by an infinite number of irreducibles.

11:00-12:00: **S. Corry**

Title: Galois branched covers of finite graphs

Abstract: Recently, M. Baker and S. Norine have suggested that the category of finite graphs with harmonic morphisms should be considered as a toy model for the category of Riemann surfaces with holomorphic maps. In this talk, I will discuss this analogy from the Galois-theoretic point of view. In particular, I will describe a Grunwald-Wang property enjoyed by finite graphs (valid for arbitrary groups), and as a corollary show that the Inverse Galois Problem has an affirmative answer in the graph-theoretic context.

13:30-14:30: Programme du jour

14:45-15:45: **T. Ichikawa**

Title: Motivic properties of modular groups

Abstract: We consider mixed motives obtained as the nilpotent completions of modular groups, especially of elliptic modular groups. We introduce Hecke operators which act on these motives and satisfy the congruence relation. Further, we use a motivic decomposition given by the Hecke operators to show the algebraicity of multiple modular L -values.

16:00-17:00: **P. Debes**

Title: The Regular Inverse Galois Problem and the Hilbert-Grunwald property

Abstract: Our main result combines a Grunwald-Wang type conclusion for arbitrary groups, a new version of Hilbert's irreducibility theorem and a p -adic form a la Harbater, but with good reduction, of the Regular Inverse Galois Problem. As a consequence we obtain a statement that questions the validity of the RIGP over \mathbb{Q} . (joint work with Nour Ghazi)

October 23 Saturday

09:45-10:45: **J. Tong**

Title: On the variation of fundamental groups

Abstract: In this talk, I will first recall a remarkable result, which is due to Tamagawa, on the variation of fundamental groups of curves in positive characteristic. Then I will discuss our main tool (the theta divisor of Raynaud) to study this problem, and a very partial improvement of the result of Tamagawa.

11:00-12:00: **A. Auel**

Title: A Milnor conjecture for p -adic curves

Abstract: Merkurjev's theorem—that every 2-torsion Brauer class is represented by the Clifford algebra of a quadratic form—is in general false when the base is no longer a field. Parimala, Scharlau, and Sridharan found complete p -adic curves for which Merkurjev's theorem is equivalent to the existence of a rational theta characteristic. We'll discuss how replacing quadratic forms by line bundle-valued quadratic forms salvages Merkurjev's theorem for p -adic curves.

13:30-14:30: Programme du jour

14:45-15:45: **G. Yamashita**

Title: p -adic multiple zeta values and motivic Galois group

Abstract: We will give an upper bound of the dimensions of the p -adic multiple L -value spaces. This upper bound comes from algebraic K -theory, via the theory of mixed Tate motives, and it's a p -adic analogue of Deligne-Goncharov's theorem. We also formulate a p -adic analogue of Grothendieck's conjecture on a special element in the motivic Galois group of the category of mixed Tate motives.

16:00-17:00: **G. Quick**

Title: Galois actions on étale homotopy types

Abstract: We will discuss a rigid model for Galois actions on étale homotopy types of varieties over a field. This allows to talk about homotopy fixed points and homotopy orbits of such actions and provides descent spectral sequences for both of them. The main applications are a generalization of Cox's theorem for varieties over arbitrary fields and a possible different perspective on the section conjecture (which is joint work with Kirsten Wickelgren).