

**ADVANCED STUDIES
IN PURE MATHEMATICS 63**

Chief Editor of the Series: Takayuki Oda (Univ. of Tokyo)

**Galois–Teichmüller Theory
and Arithmetic Geometry**

Edited by

Hiroaki Nakamura (Okayama University)

Florian Pop (University of Pennsylvania)

Leila Schneps (CNRS, University of Paris 6)

Akio Tamagawa (RIMS, Kyoto University)



Mathematical Society of Japan

Preface

From the beginning of the 1980's, Grothendieck's *Esquisse d'un Programme* triggered tremendous developments in number theory and arithmetic geometry, which aim at reformulating various difficult open problems in homotopy theoretical terms in order to have a new approach to solving them. Tackling those questions with group theoretical methods addresses various problems concerning Galois groups of number fields (and more general fields) and their representations, and extends to the studies of polylogarithms and multiple zeta values, motives, rational points on arithmetic varieties, and effectiveness questions in arithmetic geometry.

The growing interest in this new point of view is reflected in very intensive and extensive research, and the well-attended and very successful events organized around this area of investigation. The activity in October 2010 in Kyoto focused on Grothendieck's Program thematically followed the semester-long *NAG Programme* held at the Isaac Newton Institute, Cambridge, UK, from July to December 2009.

The Kyoto activity in October 2010 consisted of two parts: a one-week public workshop (October 25–30) together with a satellite closed meeting (October 19–24 participated by invited experts) prior to it. The public part was promoted by the Mathematical Society of Japan (MSJ), and both parts were set up under the auspices of the Research Institute for Mathematical Sciences (RIMS), Kyoto University, Japan.

The first part was the *Galois Theoretic Arithmetic Geometry* meeting which took place at the International Institute for Advanced Studies (IIAS) and in the Keihanna Plaza Hotel, in a suburb of Kyoto. It was designed as one of the RIMS camp-style seminars of 2010. There were forty-nine participants (including twenty-five from abroad), who shared sixteen scheduled research talks together with several elastic slots called “programme du jour” and “night sessions” for spontaneous expositions and discussions. The organizers of this meeting were Hiroaki Nakamura (Chair), Florian Pop, Leila Schneps, Akio Tamagawa and Yuichiro Hoshi.

The second part of the activity was the Joint MSJ-RIMS Conference *Development of Galois–Teichmüller Theory and Anabelian Geometry*, which took place at the RIMS in Kyoto. There were one hundred and seven participants (including thirty-eight from abroad), and twenty-two lectures, including both survey talks and research presentations. This conference was organized by Hiroaki Nakamura (Chair), Florian Pop, Leila Schneps and Akio Tamagawa. It was designed as the 3rd MSJ-SI

(Mathematical Society of Japan, Seasonal Institute) and as one of the RIMS workshops of 2010.

Whether first-time visitors to Japan or not, many overseas participants at the Kyoto activity could not but be struck by the incredible charm and beauty of the country, the warm and efficient welcome from the hotel staff, and the punctual organization. For the first part of the activity, the participants all stayed together in the Keihanna Plaza Hotel and the International Institute for Advanced Study, which meant that mathematics had already begun at the breakfast tables (where some adventurers ate Japanese style soup and pickles while others relied on coffee and other more standard European fare), and continued throughout the day until the marathon after-dinner night-sessions where anyone was welcome to pick up the chalk and start a discussion on an open problem. Slots that had been kept purposely open were filled on the spot with lectures planned to be useful introductions to later more advanced ones; special thanks are due to the impromptu speakers (P. Lochak, Y. Hoshi, P. Cartier), whose presentations were fresh, lively and informative.

For the second part of the activity, the participants were lodged in the beautiful city of Kyoto. The schedule was a full and busy one, but each day, the lunchtimes and the late afternoons saw knots of mathematicians exploring the splendid gardens and temples and enjoying the crisp, sunny autumn weather.

The lectures at the Kyoto activity covered the most up-to-date advances in the different directions that emerged from the informal but international distribution of Grothendieck's famous text "Esquisse d'un Programme" in the early 80's, in which he articulated a vast vision of the "Galois–Teichmüller theory" of our volume title. Throughout both parts of the conference, the presence of Y. Ihara—who already in the 1960's had discovered deep arithmetic phenomena of fundamental groups that were subsequently found to mesh in an extraordinary manner with some of Grothendieck's ideas in the Esquisse—was an honor and a delight, as was that of the unstoppably energetic and mathematically fertile P. Cartier.

The conference saw many lectures on progress in anabelian geometry and related arithmetic geometry, new results on Grothendieck's section conjecture; studies on the Grothendieck–Teichmüller group, including advances in the complex network of relationships between the Lie algebras connected to the subject: the Grothendieck–Teichmüller Lie algebra, the Kashiwara–Vergne Lie algebra, the double shuffle Lie algebra and multiple zeta values, and their motivic versions. More than one surprise answer to an open question came to amaze the participants; nothing more, perhaps than the discreet announcement by F. Brown

in the middle of a quiet Thursday afternoon, that he had solved the long-standing conjecture that the subcategory of mixed Tate motives generated by motivic multiple zeta values was in fact equal to the full category of mixed Tate motives over \mathbb{Z} .

The present volume *Galois–Teichmüller Theory and Arithmetic Geometry* collects twenty-four articles written by speakers (and their co-authors) at the Kyoto activity in October 2010. All the papers of the volume have been refereed by anonymous experts.

Thanks and Acknowledgements:

We would like to express our sincere gratitude to the Mathematical Society of Japan, the Research Institute for Mathematical Sciences, the International Institute for Advanced Study, the Keihanna Plaza Hotel and their secretaries/staffs for their constant and efficient assistance.

Besides the budget of the Mathematical Society of Japan for MSJ-SI and the budget of the Research Institute for Mathematical Sciences, Kyoto University for RIMS workshops/camp-style seminars, we had financial support (mainly for inviting overseas participants to the two conferences) from the following grants: NSF Grant DMS 1044746 (Florian Pop), JSPS Grants-in-Aid for Scientific Research (A)19204002 (Makoto Matsumoto), (A)22244001 (Takeshi Saito), (B)21340009 (Hiroaki Nakamura), (B)22340006 (Akio Tamagawa), JSPS/CNRS Japan-France Research Cooperative Program (Anna Cadoret and Akio Tamagawa), and Kyoto University Global COE Program (the Department of Mathematics and the Research Institute for Mathematical Sciences, Kyoto University).

July 2012

Proceedings Editors:
Hiroaki Nakamura
Florian Pop
Leila Schneps
Akio Tamagawa

All papers in this volume have been refereed and are in final form. No version of any of them will be submitted for publication elsewhere.