Spring seminar on
Arithmetic Galois theory
in Toyonaka 2025

[Version March 11, 2025]

Date: March 31-April 1st, 2025
Place: Room E404, Building E, Graduate School of Science, Osaka University, Toyonaka Campus

Organizers:
Benjamin Collas (RIMS, Kyoto University)
Pierre Dèbes (Lille University)
Hiroaki Nakamura (Osaka University)

March 31, 2025 (Mon)

10:00-11:00 Koichiro SAWADA (RIMS, Kyoto University)
Families preserving isomorphisms via techniques in anabelian geometry

11:10-12:10 Shun ISHII (Keio University)
On graded Lie algebras associated to pro-p outer Galois representations of once-punctured elliptic curves with complex multiplication

(Lunch)

14:00-15:00 Reiya TACHIHARA (RIMS Kyoto University)
Aspects of Combinatorial Anabelian Geometry

(Coffee and Cookies)

15:40-16:40 Naganori YAMAGUCHI (Institute of Science Tokyo)
Finite Step Solvable Aspects of Anabelian Geometry

April 1, 2025 (Tue)

9:30-10:30 Simon RUTARD (Nagoya University)
On values at nonpositive integer tuples of multiple zeta functions of generalized Hurwitz type

10:40-11:40 Séverin PHILIP (Stockholm University)
Oda's problem and the l-monodromy fixed fields of special loci

11:50-12:50 Nao KOMIYAMA (Osaka University)
Shuffle products for multiple zeta functions and double shuffle equations for multiple zeta values

(Lunch, Coffee, Discussions – loosely organized)

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http://www4.math.sci.osaka-u.ac.jp/~nakamura/ArithmeticDay2025/

Titles and Abstracts

Monday - March 31

10:00–11:00 **Koichiro SAWADA, RIMS Kyoto University, Japan**

Families preserving isomorphisms via techniques in anabelian geometry

For an isomorphism between closed subgroups of a profinite group G, if the image of any pro-cyclic subgroup I via this isomorphism is conjugate in G to I, then we shall say that this isomorphism is "families preserving".
Jarden and Ritter showed that, for a certain type of profinite group G –that includes the absolute Galois group of a p-adic local field and the étale fundamental group of hyperbolic curves–, every normal automorphism of G is inner. Their proof was divided into two steps: first showing that every normal automorphism is "families preserving" (in G), and then showing that every "families preserving" automorphism is inner.
In this talk, we discuss, from an anabelian geometrical point of view, whether a "families preserving" isomorphism between closed subgroups of a profinite group, such as the absolute Galois group of a certain field (Hilbertian field, Henselian discrete valuation field of positive residue characteristic) or its quotient, is induced from an inner automorphism. This is a joint work with Arata Minamide and Shota Tsujimura.

11:10–12:10 **Shun ISHII, Keio University, Japan**

On graded Lie algebras associated to pro-p outer Galois representations of once-punctured elliptic curves with complex multiplication

In this talk, we discuss structures of graded Lie algebras over $\mathbb{Z}_p$ associated to outer Galois actions on the pro-p fundamental groups of once-punctured elliptic curves with complex multiplication. We show such Lie algebras are generated by certain explicit elements, at least over $\mathbb{Q}_p$, under reasonable hypothesis. This result is an analogue of a result of R. Hain and M. Matsumoto that the graded Lie algebra associated to the pro-p outer Galois representation of the projective line minus three points is generated by Soulé elements. If time permits, we also discuss how the integral structures of these Lie algebras are related to a profinite inverse Galois-theoretic problem for pro-p groups of particular types, and to Greenberg's generalized conjecture in Iwasawa theory.

14:00–15:00 **Reiya TACHIHARA, RIMS Kyoto University, Japan**

Aspects of Combinatorial Anabelian Geometry

Combinatorial anabelian geometry is a branch of anabelian geometry that focuses mainly on combinatorial and algebro-geometric objects with arithmetic motivations. Its core of study includes the dual semi-graphs of the stable reductions of curves, the outer actions of the absolute Galois group of the field of Laurent power series over the complex numbers, and the outer automorphism group of the profinite braid group, among others.
In this talk, I will present some basic notions and fundamental results of combinatorial anabelian geometry, along with its classical application to the faithfulness of the outer Galois action in arithmetic anabelian geometry. I will also briefly mention my own results on generalizations of the combinatorial Grothendieck conjecture, which is one of the most fundamental results in combinatorial anabelian geometry.

15:40–16:40 **Naganori YAMAGUCHI, Institute of Science Tokyo, Japan**

Finite Step Solvable Aspects of Anabelian Geometry

Anabelian geometry has a fundamental conjecture by A. Grothendieck, which states that the geometric properties of (algebraic) hyperbolic curves can be characterized group-theoretically via their arithmetic fundamental groups. This conjecture has been proved for finitely generated fields over the rational field by H. Nakamura, A. Tamagawa, and S. Mochizuki. In this talk, we focus on a related open problem known as the finite-step solvable Grothendieck conjecture, which concerns the reconstruction of the geometric properties of hyperbolic curves from the maximal geometrically finite-step solvable quotient of their arithmetic fundamental groups. We will discuss the formulation of this conjecture and outline part of its proof. If time permits, we will also explain an extension of the (finite-step solvable) Grothendieck conjecture to orbicurves.

Tuesday - April 1

9:30–10:30 **Simon RUTARD, Nagoya University, Japan**

On values at nonpositive integer tuples of multiple zeta functions of generalized Hurwitz type

Multiple zeta functions of generalized Hurwitz type encompass a broad class of zeta functions, including zeta functions associated with root systems, and Euler-Zagier multiple zeta functions. While these functions extend meromorphically to the entire complex space, nonpositive integer tuples typically correspond to points of indeterminacy. To address this, Y. Komori (2008) introduced directional values at such points, which he defined via limits along fixed directions. He also established a formula for these values involving non-explicit generalized Bernoulli numbers. In this talk, we investigate the analytic properties of certain multiple zeta functions of generalized Hurwitz type and establish explicit formulas for both their directional values and directional derivative values at nonpositive integers.

10:40–11:40 **Séverin PHILIP, Stockholm University, Sweden**

Oda's problem and the ℓ-monodromy fixed fields of special loci

In this talk I will first present the classical construction of the ℓ-monodromy fixed fields of curves and their moduli spaces leading to Oda's problem. I will then show an approach to this problem using the cyclic special loci. It will start with a focus on the relations between the different moduli spaces involved, and their ℓ-monodromy fixed fields followed by an overview of the main step which is an application of the maximal degeneration method of Ihara-Nakamura using Matsumoto-Seyama curves.

11:50–12:50 **Nao KOMIYAMA, Osaka University, Japan**

Shuffle products for multiple zeta functions and double shuffle equations for multiple zeta values

Shuffle products and harmonic products play an important role in the study of MZVs, and it is known that harmonic products extend naturally to functional equations of multiple zeta functions. On the other hand, Komori, Matsumoto, and Tsumura gave an extension of shuffle products to functional equations using zeta functions of root systems. By combining their functional equations with ones for harmonic products, they showed that the double shuffle relations of MZVs follow. In this talk, we extend their functional equations and discuss a relationship with the extended double shuffle relations of MZVs. We also show that several known functional relations follow from our functional equations. This is a joint work with Takeshi Shinohara.