

# On Kashiwara-Vergne Lie algebra and double shuffle Lie algebra in mould theory

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## Abstract

In 2012, Schneps ([6]) showed that there exists an embedding between the double shuffle Lie algebra introduced by Racinet ([4]) and the Kashiwara-Vergne Lie algebra introduced by Alekseev and Torossian ([1]). In the proof of this embedding, a relation called the *senary relation* is used, which is a notion introduced in Ecalle's study ([2]) of multiple zeta values using mould theory. On Lie algebras and embeddings above, bigraded or elliptic versions have also been studied ([3], [5]). In my talk, I will explain the above topics and recent topic ([7]) as much as time permits.

## References

- [1] A. Alekseev and C. Torossian, The Kashiwara-Vergne conjecture and Drinfeld's associators, *Ann. Math.*, 175 (2012), no. 2, p. 415-463.
- [2] J. Ecalle, The flexion structure and dimorphy: flexion units, singulators, generators, and the enumeration of multizeta irreducibles, *Asymptotics in dynamics, geometry and PDEs. Generalized Borel summation. Vol. II*, 2011, p. 27-211.
- [3] H. Furusho and N. Komiyama, Kashiwara-Vergne and dihedral bigraded Lie algebras in mould theory, *Ann. Fac. Sci. Toulouse Math.*, (6) 32 (2023), no. 4, 655-725.
- [4] G. Racinet, Doubles mélanges des polylogarithmes multiples aux racines de l'unité, *Publ. Math. Inst. Hautes Études Sci.*, No. 95 (2002), 185-231.
- [5] E. Raphael and L. Schneps, On linearised and elliptic versions of the Kashiwara-Vergne Lie algebra, 2017, <https://arxiv.org/abs/1706.08299v1>.
- [6] L. Schneps, Double shuffle and Kashiwara-Vergne Lie algebras, *J. Algebra* 367, 2012, p. 54-74.
- [7] L. Schneps, The double shuffle Lie algebra injects into the Kashiwara-Vergne Lie algebra, 2025, <https://arxiv.org/abs/2504.14293>.