Hayama Symposium on Complex Analysis in Several Variables XXII

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ABSTRACTS

Chen-Yu Chi (National Taiwan University)
Pseudonorms and p-adic birational Torelli-type theorem

For a complex projective manifold X, the pseudonormed m-canonical space $\left(\Gamma(X, K_X^{\otimes m}), \langle\!\langle\cdot\rangle\!\rangle_m\right)$ is a fundamental birational invariant of X. Roughly speaking, the birational Torelli type theorem says that, for complex projective manifolds X and X' of general type, any isometry between their pseudonormed m-canonical spaces for one sufficiently large m comes from a birational equivalence between X and X'. In this talk we will talk about a p-adic version of the birational Torelli type theorem.

Fusheng Deng (University of Chinese Academy of Sciences)
Linear invariants of bounded domains

We show that two hyperconvex bounded domains are biholomorphically equivalent if and only if their I^p holomoprhic function spaces are linearly isometric as abstract metric linear spaces, for some $0 < p < 2$. This is based on a work joint with Zhiwei Wang, Liyou Zhang, and Xiangyu Zhou.

Tien-Cuong Dinh (National University of Singapore)
Random walks in $SL_2(\mathbb{C})$

We consider products of N randomly chosen 2×2 matrices. The aim of the theory is to study statistical properties of these products when N goes to infinity. By introducing a new method using complex analysis, we prove several equidistribution properties and limit theorems. The talk is based on my joint work with Lucas Kaufmann and Wu Hao.

Jun-Muk Hwang (Institute for Basic Science)
Geometry of Elie Cartan's "Pfaffian systems in 5 variables"

Elie Cartan's celebrated paper "Pfaffian systems in 5 variables" in 1910 studied the equivalence problem for general Pfaffian systems of rank 3 in 5 variables as the curved version of the Pfaffian system with G_2 symmetry. The G_2 case admits a beautiful geometric correspondence with Engel's PDE system. We discuss a recent joint work with Qifeng Li, where we generalize this correspondence to the curved case of general Pfaffian systems of rank 3 in 5 variables. Our approach combines ideas from geometric control theory and algebraic geometry.

Hisashi Kasuya (Osaka University)
Hodge theory on Oeljeklaus-Toma manifolds —number theory, complex geometry and group symmetry—

Oeljeklaus and Toma introduce compact complex manifolds (OT-manifolds) associated with algebraic number fields. OT-manifolds do not admit Kähler metrics. We study Hodge theoretical properties of OT-manifolds by using Lie group techniques (solvmanifolds and Cousin groups).

Sung-Yeon Kim (Korea Institute for Advanced Study)
Totally geodesic isometric discs in bounded symmetric domains

Let Ω and Ω' be two bounded symmetric domains. In this talk, we investigate the properties of totally geodesic isometric embedding $f : \Omega \to \Omega'$ with respect to Bergman/Kobayashi metric. In particular, we discuss the question of holomorphicity of sufficiently smooth totally geodesic isometric disc $f : \Delta \to \Omega$. As an application, we give a sufficient condition for a totally geodesic isometric embedding $f : \Omega \to \Omega'$ to be holomorphic or anti-holomorphic. This is a joint work with Aeryeong Seo.

Yuta Kusakabe (Kyoto University)
Oka manifolds and the dual Levi problem

A complex manifold is an Oka manifold if continuous maps from Stein manifolds can be deformed into holomorphic maps with approximation and interpolation. Dual to Stein manifolds, which are the most natural sources of holomorphic maps, Oka manifolds are the most natural targets. On the other hand, the Levi problem asks for the geometric characterization of Stein domains by pseudoconvexity. In this talk, we discuss the "dual" Levi problem which asks for the geometric characterization of Oka domains.

Finnur Lárusson (University of Adelaide)
A strong parametric h-principle for complete minimal surfaces

I will describe new joint work with Antonio Alarcón (arXiv:2106.03495). We prove a parametric h-principle for complete nonflat conformal minimal immersions of an open Riemann surface M into $\mathbb{R}^n$, $n \geq 3$. It follows that the inclusion of the space of such immersions into the space of all nonflat conformal minimal immersions is a weak homotopy equivalence. When M is of finite topological type, the inclusion is a genuine homotopy equivalence. By a parametric h-principle due to Forstnerič and Lárusson, the space of complete nonflat conformal minimal immersions therefore has the same homotopy type as the space of continuous maps from M to the punctured null quadric. Analogous results hold for holomorphic null curves $M \to \mathbb{C}^n$ and for full immersions in place of nonflat ones.

Sheng Rao (Wuhan University)
Invariance of plurigenera and Chow-type lemma

I will report my recent joint work with I-Hsun Tsai, which answers a question of Demailly whether a smooth family of nonsingular projective varieties admits the deformation invariance of plurigenera affirmatively, and proves this more generally for a flat family of varieties with only canonical singularities and uncountable ones therein of general type and also two Chow-type lemmata on the structure of family of projective varieties.

Ryosuke Takahashi (Kyushu University)
Some geometric flow approaches for the deformed Hermitian–Yang–Mills equation

On SYZ mirror symmetry, a deformed Hermitian Yang–Mills (dHYM) metric is a fiber metric on a holomorphic line bundle, which is the mirror object to a special Lagrangian section of the dual torus fibration. As a parabolic analogue, Jacob and Yau introduced the Line Bundle Mean Curvature Flow (LBMCF) as the mirror of the Lagrangian Mean Curvature Flow. In this talk, we explore some geometric flow approaches for dHYM metrics as follows: (A) On Kähler surfaces, it is known that the existence of dHYM metrics is equivalent to a Kähler condition for a certain cohomology class. We relax this condition and study how the LBMCF blows up. (B) Recently, Collins and Yau discovered a new variational characterization for dHYM metrics. Motivated by this, we introduce a new geometric flow which is designed to deform a given metric to a dHYM one. Then we show that this new flow potentially has more global existence and convergence properties than the LBMCF.

Kaushal Verma (Indian Institute of Science)
Limits of an increasing sequence of complex manifolds

Fix a domain D and consider an increasing sequence of domains D_j each of which is biholomorphic to D. In their '81 paper, Fornaess and Sibony studied the question of identifying (up to a biholomorphism) the possible limits of the union of the D_j's. Call this union D_∞. Under the hypothesis that $D/\mathrm{Aut}(D)$ is compact, and D_∞ is either Kobayashi hyperbolic or has corank one, they were able to identify D_∞. In the latter case, the role of retracts is essential—and when D is either the ball or a polydisc, a much better understanding of D_∞ is possible.

The purpose of this talk will be to revisit and explore this theme by allowing various other domains D that do not admit a compact quotient under $\mathrm{Aut}(D)$. The main result is an identification of D_∞ in such cases. When D is the product of a disc with the ball in $\mathbb{C}^n$, retracts are identified completely and this leads to a description of D_∞ when the Kobayashi metric on it has corank one.

This is joint work with G P Balakumar, Diganta Borah and Prachi Mahajan.

Xiaokui Yang (Tsinghua University)
Strictly nef vector bundles and characterization of projective spaces

In this talk, we will present some recent progress on the geometry of strictly nef vector bundles, with focus on the characterization of projective spaces.

Toru Yoshiyasu (Research Institute for Mathematical Sciences, Kyoto University)
Stabilized convex symplectic manifolds are Weinstein

It is known that a Weinstein manifold, a symplectic counterpart of a Stein manifold, is of the homotopy type of a half-dimensional CW-complex and such an even-dimensional manifold admits a Weinstein structure. On the other hand, it was completely unknown whether such a convex symplectic manifold is Weinstein. I will explain a partial answer to this problem. More precisely, I will show that a stabilized convex symplectic manifold with the homotopy type of a half-dimensional CW-complex is symplectomorphic to a Weinstein manifold. This is a joint work with Yakov Eliashberg and Noboru Ogawa.

Langfeng Zhu (Wuhan University)
Generalizations of Siu's lemma and applications

In this talk, we discuss some generalizations of Siu's lemma and their applications to extension theorems with singular weights on Kähler manifolds. We also discuss their applications to comparison between singular metrics on twisted relative pluricanonical bundles, and subadditivity of Kodaira–Iitaka's dimensions. These are joint works with Professor Xiangyu Zhou.

Abstracts for Short Communications

Naveen Gupta (University of Delhi)
Squeezing function and its generalisations

We discuss some aspects of the theory arising by changing unit ball to other model domains in the definition of squeezing function. To begin with we take this model domain as polydisc. We will then discuss our work on dbalanced squeezing function, where we have taken the model domain as any dbalanced domain (which is more general than the work done by Rong and Yang in this direction). We will also see the relation between these squeezing functions and the corresponding Fridman invariants as well in each case.

Eiji Inoue (RIKEN)
Perelman's entropy in Kähler geometry

Perelman's entropy was originally introduced to study the non-collapsing result along Ricci flow. Apart from this context, I recently discovered a special aspect of Perelman's entropy in Kähler geometry. It is connected to the existence of a canonical metric on a polarized manifold, beyond Kähler-Ricci soliton, and is also connected to a variant of K-stability. Among many fascinating aspects, I would like to focus on a result on an algebro-geometric lower bound on Perelman's entropy, which I believe is optimal.

Takeo Ohsawa (Nagoya University)
L^2 cohomology with weights and bundle convexity of certain locally pseudoconvex domains

Let X be a reduced complex space and let G be a holomorphic vector bundle. X is said to be G-convex if, for any compact set $K/subset X$ one can find a compact set $\hat{K} \subset X$ such that the following holds for any $x \in X \setminus \hat{K}$:

> For any neighborhood U of the zero section of G and for any point P in the fiber G_x, there exists a holomorphic section s of G satisfying $s(x) = P$ and $s(K) \subset U$.

A recent result on the bundle convexity in this sense for some class of locally pseudoconvex domains will be reported.