LIE ALGEBRA ACTIONS ON MODULAR FORMS

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Abstract. We report on the dictionary for passing between statements about classical differential operators for automorphic forms on the Siegel upper half plane and statements about the action of the Lie algebra of $\mathrm{Sp}_n(\mathbb{R})$ on automorphic forms on the group $\mathrm{Sp}_n(\mathbb{R})$ (or ∞-components of adelic automorphic forms) and use this to replace some results about differential operators and restrictions of automorphic forms and differential operators in [1].

1. The dictionary

There is a well known correspondence between automorphic forms on the Siegel upper half space $\mathfrak{H}_n \subseteq M_n^{\mathrm{sym}}(\mathbb{C})$ and automorphic forms on the symplectic group $\mathrm{Sp}_n(\mathbb{R})$ (more generally: automorphic forms on a real semisimple Lie group and automorphic forms in its symplectic space).

As is also well known this correspondence may be extended to a correspondence between invariant differential operators on automorphic forms on $\mathfrak{H}_n$ and the action of the Lie algebra $\mathfrak{sp}_n$ and its complexification $(\mathfrak{sp}_n)_{\mathbb{C}}$ on the group theoretic automorphic forms.

We follow Harris [3, 4] in describing this correspondence, for the classical side we refer to the article [7] of Yamauchi in these proceedings and the references to Shimura's work given there.

We use the usual notations:

The Siegel upper half plane of degree n is

$$\mathfrak{H}_n = \{X + iY \in M_n^{\mathrm{sym}}(\mathbb{C}) \mid X, Y \in M_n^{\mathrm{sym}}(\mathbb{R}),\ Y > 0\},$$

on $\mathfrak{H}_n$ we have the action of matrices $\gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}_n(\mathbb{R}) =: G$ (with $A, B, C, D \in M_n(\mathbb{R})$) by

$$\gamma Z := (AZ + B)(CZ + D)^{-1}.$$

For $\gamma = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in G$ as above and $Z \in \mathfrak{H}_n$ we have the factor of automorphy

$$j(\gamma, Z) = CZ + D \in \mathrm{GL}_n(\mathbb{C}).$$

For a rational representation $\rho: \mathrm{GL}_n(\mathbb{C}) \longrightarrow \mathrm{Aut}(V)$, where V is a finite dimensional complex vector space, and an arithmetic subgroup Γ of $\mathrm{Sp}_n(\mathbb{R})$ a C^∞-function $f: \mathfrak{H}_n \longrightarrow V$ is called Γ-automorphic of type ρ if $f(\gamma Z) = \rho(j(\gamma, Z))f(Z)$ holds for all $\gamma \in \Gamma$ and f is of moderate growth.

If f as above is holomorphic (and in case $n = 1$ holomorphic in the cusps of Γ) it is a modular form of type ρ.

Definition and Lemma 1.1. *Let V be a finite dimensional $\mathbb{C}$-vectorspace, $\rho: \mathrm{GL}_n(\mathbb{C}) \longrightarrow \mathrm{Aut}(V)$ a rational representation, $f: \mathfrak{H}_n \longrightarrow V$ a C^∞-function. Define $\Phi_{\rho,f}: \mathrm{Sp}_n(\mathbb{R}) \longrightarrow V$ by*

$$\Phi_{\rho,f}(g) := \rho(j(g,i))^{-1} f(gi1_n).$$

Then for $A, B \in \mathrm{GL}_n(\mathbb{R})$ with $u = A + Bi \in U_n(\mathbb{C})$ and $r(u) := \begin{pmatrix} A & -B \\ B & A \end{pmatrix} \in \mathrm{Sp}_n(\mathbb{R})$ one has

$$\Phi_{\rho,f}(g \cdot r(u)) = \rho(u)^{-1} \Phi_{\rho,f}(g) \tag{1.1}$$

for all $g \in \mathrm{Sp}_n(\mathbb{R})$.

C^∞-functions $\Phi: \mathrm{Sp}_n(\mathbb{R}) \longrightarrow V$ with the property in (1.1) are said to be of type ρ, the space of all such functions is denoted by

$$C^\infty(G, V)_\rho.$$

If in addition $\Gamma \subseteq \mathrm{Sp}_n(\mathbb{R})$ is an arithmetic subgroup and $f(\gamma Z) = \rho(j(\gamma, Z))f(z)$ holds for all $\gamma \in \Gamma$, $Z \in \mathfrak{H}_n$ the function $\Phi_{\rho,f}$ satisfies $\Phi_{\rho,f}(\gamma, g) = \Phi_{\rho,f}(g)$ for all $\gamma \in \Gamma$, we write then $\Phi_{\rho,f} \in C^\infty(\Gamma \setminus G, V)_\rho$.

The correspondence $f \longrightarrow \Phi_{\rho,f} \in C^\infty(G, V)$ can be inverted, associating a C^∞-function $f_{\Phi,\rho}: \mathfrak{H}_n \longrightarrow V$ to $\Phi \in C^\infty(G, V)_\rho$.

Lemma 1.2. a) *Let $\mathfrak{g} = \left\{ \begin{pmatrix} A & B \\ C & -{}^tA \end{pmatrix} \in \mathrm{GL}_{2n}(\mathbb{R}) \mid A \in M_n(\mathbb{R}),\ B, C \in M_n^{\mathrm{sym}}(\mathbb{R}) \right\}$ be the Lie algebra of G and $\mathfrak{g}_\mathbb{C}$ its complexification. Then*

$$\mathfrak{g}_\mathbb{C} = \mathfrak{k}_\mathbb{C} \oplus \mathfrak{p}_+ \oplus \mathfrak{p}_-$$

where

$$\begin{aligned} \mathfrak{p}_+ &= \left\{ \begin{pmatrix} A & iA \\ iA & -A \end{pmatrix} = p_+(A) \mid A \in M_n^{\mathrm{sym}}(\mathbb{C}) \right\} \\ \mathfrak{p}_- &= \left\{ \begin{pmatrix} A & -iA \\ -iA & -A \end{pmatrix} = p_-(A) \mid A \in M_n^{\mathrm{sym}}(\mathbb{C}) \right\} \\ \mathfrak{k}_\mathbb{C} &= \left\{ \begin{pmatrix} A & -iB \\ iB & A \end{pmatrix} = p_0(A, B) \mid {}^tA = -A,\ {}^tB = B \right\} \end{aligned}$$

b) *Let $n = 1$ in the above setup, $\rho = \det^k$, for $u = e^{i\theta} \in U(1)$ write*

$$r(u) = r(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix},$$

let $\Phi = \Phi_{f,\rho} = \Phi_{f,k} \in C_\infty(G, \mathbb{C})_\rho = C_\infty(G, \mathbb{C})_k$ be as in (1.1). Put

$$\begin{aligned} H &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \in \mathfrak{k}_{\mathbb{C}}, \\ X &= \frac{1}{2}\begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \in \mathfrak{p}_+, \\ Y &= \frac{1}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \in \mathfrak{p}_-. \end{aligned}$$

Let $A \in \mathfrak{g}$ act on Φ as usual by

$$A * \Phi(g) := \frac{d}{dt}(\Phi(g \exp(tA))) \mid_{t=0}$$

and extend this action linearly to $\mathfrak{g}_{\mathbb{C}}$.

Then one has

$$\begin{aligned} X * \Phi &\in C^\infty(G, \mathbb{C})_{k+2} \\ Y * \Phi &\in C^\infty(G, \mathbb{C})_{k-2} \\ H * \Phi_f &= k \cdot \Phi_f. \end{aligned}$$

c) *If f is a Γ-automorphic form of weight k, the function*

$$D_k f := f_{X*\Phi, k+2}$$

is Γ-automorphic of weight $k+2$ and

$$E_k f := f_{Y*\Phi, k-2}$$

is Γ-automorphic of weight $k-2$.

One has $D_k f(x+iy) = 2i\frac{\partial f}{\partial z} + \frac{k}{y} f = -4\pi\delta_k f$, where δ_k is the Maaß operator [5, 3] *and*

$$E_k f(x+iy) = 2iy^2 \frac{\partial f}{\partial \bar{z}}.$$

In particular, f as above is holomorphic if and only if $Y * \Phi_f = 0$.

Proof. The proofs of Lemma 1.1 and 1.2 are well known and easily checked, see [3]. □

Corollary 1.3. *Let f be a C^∞-automorphic form of weight k on $\mathfrak{H}_1 = \mathfrak{H}$ for the congruence subgroup $\Gamma \subseteq \mathrm{SL}_2(\mathbb{Z}) = \mathrm{Sp}_n(\mathbb{Z})^*$ which is holomorphic in the cusps of Γ. Then*

a) *f is nearly holomorphic (see* [7]*) if and only if $Y^r\Phi_f = 0$ for some $r \in \mathbb{N}$.*

b) *Equivalently, f is nearly holomorphic if and only if the $U(\mathfrak{g}_{\mathbb{C}})$-module generated by f contains a function $\tilde{\Phi}$ of holomorphic type (where $U(\mathfrak{g}_{\mathbb{C}})$ is the universal enveloping algebra).*

c) *If this is the case and the $U(\mathfrak{g}_{\mathbb{C}})$-module generated by Φ_f contains no nonzero constants, it is a sum of cyclic modules generated by functions Φ_j of holomorphic type.*

Proof. a) and b) are obvious consequences of 1.1 and 1.2.

Concerning c), assume that $Y^r * \Phi_f = 0$ but $Y^{r-1} * \Phi_f \neq 0$ with $\Psi_i = Y^{r-1} * \Phi_f$ is not constant. The function Ψ corresponds then to a holomorphic modular form of some weight $k > 0$ and we have $Y * X * \Psi = c'\Psi_1$ with $c' \neq 0$ and iterating we find that $\Phi_1 := X^{r-1}\Psi_1$ satisfies $Y^{r-1}\Phi_1 = c_1\Psi_1 = c_1 Y^{r-1}\Phi$ with $c \neq 0$. Replacing Φ by $\Phi - \frac{1}{c_1}\Phi_1$ we find recursively functions Φ_j and constants $c_j \neq 0$ so that $Y^{r-1}(\Phi - \sum_{i=1}^{j} \frac{1}{c_j}\Phi_j) = 0$ and such that $\Psi_j : -Y^{r-j}\Phi_1$ is of holomorphic type and $\Phi_j = X^{r-j}\Psi_j$ holds.

Φ (and hence $U(g)\Phi$) is therefore in the same of the cyclic $U(\mathfrak{g})$-modules generated by the Ψ_j. ☐

In order to formulate the analogue of the results for the case $n = 1$ we recall the following notations from [7]:

Put $T = M_n^{\mathrm{sym}}(\mathbb{C})$ and consider some fixed irreducible rational representations $\rho :\ \mathrm{GL}_n(\mathbb{C}) \longrightarrow \mathrm{Aut}(V)$ as in Definition and Lemma 1.

For $u \in T$ and $f \in C^\infty(\mathfrak{H}_n, V)$ put

$$D_u f = \sum_{1 \le i \le j \le n} u_{ij} \frac{\partial f}{\partial z_{ij}}.$$

Moreover, for $Z \in \mathfrak{H}_n$ put

$$\eta := 2\,\mathrm{Im}(Z)$$

and define an operator

$$D_\rho :\ C^\infty(\mathfrak{H}_n, V) \longrightarrow C^\infty(\mathfrak{H}_n, \mathrm{Hom}(T, V))$$

by

$$((D_\rho f)(Z))(u) = \rho(\eta)^{-1}(D_u(\rho(\eta)f)(Z))$$

Finally denote by (τ, W_τ) the representation $\mathrm{Sym}^2(\mathrm{GL}_n(\mathbb{C}))$ and by $\pi = \tau^*$ the contragredient of τ.

Lemma 1.4. a) *Let $f \in C^\infty(\mathfrak{H}_n, V)$, let $p_\pm :\ T \longrightarrow \mathfrak{p}_\pm$ be as in Lemma 1.2.*

Then for $u \in T$, $g \in G$ one has with nonzero constants c_n, c_n'

$$\begin{aligned}(p_+(u) * \Phi_{f,\rho}(g) &= c_n(\Phi_{D_\rho f, \rho\otimes\tau}(g)(u),\\ (p_-(u) * \Phi_{f,g}(g) &= c_n'(\Phi_{Ef,\rho\otimes\pi}(g))(u),\end{aligned}$$

where the action of $\rho \otimes \tau$ resp. $\rho \otimes \pi$ on $\mathrm{Hom}(T, V)$ *is given by*

$$((\rho \otimes \tau)(a)(h))(u) = \rho(a)h({}^t aua)$$
$$((\rho \otimes \pi)(a)(h))(u) = \rho(a)h(a^{-1}u^t a^{-1}).$$

b) *If $f \in C_\infty(\mathfrak{H}_n, V)$ is automorphic of type ρ it is holomorphic if and only if*

$$p_-(T) * \Phi_{f,\rho} = 0.$$

Functions $\Phi : G \longrightarrow V$ with this property are called functions of holomorphic type.

c) *If $f \in C^\infty(\mathfrak{H}_n, V)$ is automorphic of type ρ for some congruence subgroup Γ then f is nearly holomorphic of degree p if and only if*

$$A_1 * \cdots * A_{p+1} * \Phi_{f,\rho} = 0$$

for all $A_1, \ldots, A_{p+1} \in \mathfrak{p}_-$.

Proof. Using the results from [7], b) and c) are immediate consequences of a).

To prove a) one has to use the chain rule to transfer the computations from f to $\Phi_{f,\rho}$; this gives

$$(p_+(u)F)(g) = c_n D_{u'} f(gi) \text{ with } u' =^t j(g,i)uj(g,i)$$

with a nonzero constant c_n. The rest of the computations is of the same type as in [6]; one has to use that for $g \in G$ one has

$$\eta(gi) = j(g,i)^t (j\overline{(g,i)})^{-1}.$$

□

Remark. *a) Contrary to the situation for $n = 1$ it seems not to be clear under which precise conditions on the $U(\mathfrak{g}_\mathbb{C})$-module generated by $\Phi_{f,g}$ it follows that this module is a sum of cyclic modules $\mathcal{V}_j$ generated by functions Ψ_j of holomorphic type. It is known (see* [7]*) that this is the case if $\rho = \det^k \otimes \rho_0$ with ρ_0 polynomial and k sufficiently large (depending on ρ_0).*

b) We see that application of the invariant differential operators coming from the $\mathfrak{p}_+$-part of $\mathfrak{g}_\mathbb{C}$ sends functions on the group of type ρ to functions of type $\rho \otimes \tau$. Iterating this as in [7] *we see that application of an element $A_1 \cdots A_k$ with $A_j \in \mathfrak{p}_+$ of the universal envoloping algebra $U(g_\mathbb{C})$ sends function on the group of type ρ fo functions of type $\rho \otimes \mathrm{Sym}^k(T)$ and consequently to functions of type $\rho \otimes \sigma$, where (W_σ, σ) is any $\mathrm{GL}_n(\mathbb{C})$-subrepresentation inside $\mathrm{Sym}^k\Gamma$.*
Taking the special case of $(\det)^2 \subseteq \mathrm{Sym}^n(T)$ one obtains (up to a nonzero constant) the classical Maaß-operator from n-fold iteration of

the operator attached to

$$\frac{1}{2}p_+\left(\begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & \ddots & & & 1 \\ \vdots & & \ddots & & \vdots \\ 1 & & & \ddots & 1 \\ 1 & 1 & \cdots & 1 & 1 \end{pmatrix}\right) \qquad \textit{(changing } \rho \textit{ to } \rho\otimes\det^2\textit{)}$$

and the operator from [2] *for the invariant differential operator attached to* $p_+(\alpha)$ *with* α *as above in the case* $k=1$.

2. An application

In his article [1] in these proceedings, Böcherer discusses (nonholomorphic) differential operators that send holomorphic functions on the upper half plane $\mathfrak{H}_n$ to functions, whose restrictions to $\mathfrak{H}_p\times\mathfrak{H}_q$ (with $p+q=n$) is holomorphic.

In the case $n=2$, he has the following example

Example (Böcherer, [1]**):** Let $k\in\frac{1}{2}\mathbb{N}_0\setminus\{0,\frac{1}{2}\}$, denote by $\delta_k^{(2)}$, $\delta_k^{(1)}$ respectively the Maaß operator for automorphic forms of weight k on $\mathfrak{H}_2$ resp. $\mathfrak{H}_1$. Then

$$(\delta_k^{(2)}-\frac{k-\frac{1}{2}}{k}(\delta_k^{(1)}\otimes\delta_k^{(1)}))|_{z_{12}=0}=(\frac{1}{2k}\partial_{11}\partial_{22}-\partial_{12}^2)_{z_n}=0,$$

in particular, this operator sends holomorphic automorphic forms of weight k on $\mathfrak{H}_2$ to holomorphic automorphic forms of weight $(k+2,k+2)$ on $\mathfrak{H}_1\times\mathfrak{H}_1$.

Since the operators $\delta_k^{(2)}$, $\delta_k^{(1)}$ and $\delta_k^{(1)}\otimes\delta_k^{(1)}$ are Lie theoretic operators in the sense discussed above, we can view this phenomenon in the Lie algebra setup as follows:

Proposition 2.1. *Let* $\mathfrak{g}$ *be the Lie algebra of* $\mathrm{Sp}_2(\mathbb{R})$ *and* $\mathfrak{g}_{\mathbb{C}}$ *its complexification. Consider the following elements of* $\mathfrak{g}_{\mathbb{C}}$*:*

$$X_1 = \frac{1}{2}\begin{pmatrix} 1 & 0 & i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}p_+\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$X_2 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & i \\ 1 & 0 & i & 0 \\ 0 & i & 0 & -1 \\ i & 0 & -1 & 0 \end{pmatrix} = \frac{1}{2}p_+\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X_4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & -1 \end{pmatrix} = \frac{1}{2}p_+\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Consider the embeddings i_1, i_2 *of the complexified Lie algebra* $\tilde{\mathfrak{g}}_{\mathbb{C}}$ *of* $\mathrm{SL}_2(\mathbb{R})$ *into* $\mathfrak{g}_{\mathbb{C}}$ *given by*

$$i_1\begin{pmatrix} a & b \\ c & -a \end{pmatrix} = \begin{pmatrix} a & 0 & b & 0 \\ 0 & 0 & 0 & 0 \\ c & 0 & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$i_2\begin{pmatrix} a & b \\ c & -a \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & 0 & b \\ 0 & 0 & 0 & 0 \\ 0 & c & 0 & -a \end{pmatrix}$$

and the corresponding embedding

$$i: \mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R}) \longrightarrow \mathrm{Sp}_2(\mathbb{R})$$

given by

$$i\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}\right) = \begin{pmatrix} a & 0 & b & 0 \\ 0 & a' & 0 & b' \\ c & 0 & d & 0 \\ 0 & c' & 0 & d' \end{pmatrix}.$$

a) *If* f *is an automorphic form of weight* k *on* $\mathfrak{H}_2$*, then*

$$(4X_1X_4 - X_2^2) * \Phi_{f,k} = (4\pi)^2 \Phi_{\delta_k^{(2)} f, k+2}$$

and (with $X \in \tilde{g}_{\mathbb{C}}$ *as in Lemma 2)* $i_1(2X) + i_2(2X)$ *sends*

$$\Phi_{f,k} | i(\mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{C}))$$

to the function on $\mathfrak{H}_1 \times \mathfrak{H}_1$ *corresponding to*

$$(4\pi)^2 (\delta_k^{(1)} \otimes \delta_k^{(1)})(f|_{\mathfrak{H}_1 \times \mathfrak{H}_1}).$$

Moreover,

$$4(X_1X_4 * \Phi_{f,k})|_{i(\mathrm{SL}_2(\mathbb{R})\times\mathrm{SL}_2(\mathbb{R}))}$$
$$= (i_1(2X) + i_2(2X)) * (\Phi_{f,k}|_{i(\mathrm{SL}_2(\mathbb{R})\times\mathrm{SL}_2(\mathbb{R}))})$$

b) *Put* $Z := \frac{2}{k}X_1X_4 - X_2^2 \in U(\mathfrak{g}_{\mathbb{C}})$ *and let* f *be as in a). Then*

$$(4\pi)^{-2}(Z * \Phi_{f,k})|_{\mathrm{SL}_2(\mathbb{R})\times\mathrm{SL}_2(\mathbb{R})}$$

is the function on G *corresponding to the function*

$$\delta_k^{(2)} f|_{\mathfrak{H}_1\times\mathfrak{H}_1} - (1-\frac{1}{2k})(\delta_k^{(1)}\otimes\delta_k^{(1)})(f|_{\mathfrak{H}_1\times\mathfrak{H}_1})$$

and $Z * \Phi_{f,k}$ *is of holomorphic type.*

Proof. We check that $(Z * \Phi_{f,k})|_{\mathrm{SL}_2(\mathbb{R})\times\mathrm{SL}_2(\mathbb{R})}$ is annihilated by both $i_1(Y)$, $i_2(Y)$; by Lemma 1.2 this proves the assertion.

For this we let the universal enveloping algebra $U(\mathfrak{g}_{\mathbb{C}})$ act on functins from the left (i. e., the rightmost factor of a product acts first) and compute the commutator

$$[i_1(Y), Z] \quad \text{in} \quad U(\mathfrak{g}_{\mathbb{C}}).$$

A routine computation gives

$$[i_1(Y), X_1X_4] = 2X_4 i(H_1), \quad [i_1(Y), X_2^2] = -2X_2[i_1(Y), X_2] - 4X_4.$$

We notice that

$$[i_1(Y), X_2] = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & i \\ -1 & 0 & i & 0 \\ 0 & -i & 0 & 1 \\ -i & 0 & -1 & 0 \end{pmatrix} \in \mathfrak{k}_{\mathbb{C}}$$

holds, and a direct computation shows that the action of this Lie algebra element annihilates $\Phi_{f,k}$: if we write this element as $K_1 + iK_2$ with real K_1, K_2 we see that for $i = 1, 2$ and $t \in \mathbb{R}$ the matrix $\exp(tK_i)$ is of the form $r(u)$ (notation as in Lemma 1.1) with $u \in U_n(\mathbb{C})$ of determinant 1 independent of t.
Moreover, $i(H_1)$ acts as multiplication by k on $C^\infty(G, \mathbb{C})_k$.

Taken together we find that $[i_1(Y), Z]$ acts as 0 on $\Phi_{f,k}$ and hence

$$i(Y) * (Z * \Phi_{f,k}) = Z * (i_1(Y) * \Phi_{f,k}) = 0,$$

the latter inequality holding since $i_1(Y) \in \mathfrak{p}_-$ annihilates the function $\Phi_{f,k}$, which is of holomorphic type.

This shows that

$$(Z * \Phi_{f,k})|_{i(\mathrm{SL}_2(\mathbb{R})\times\mathrm{SL}_2(\mathbb{R}))}$$

is of holomorphic type with respect to the first variable.

An analogous computation for $i_2(Y)$ shows that it is of holomorphic type with respect to the other variable as well. □

Remark. *It should be possible to prove more general statements about holomorphic restriction of invariant differential operators in the same way.*

References

[1] Böcherer, Siegfried: These Proceedings.
[2] Böcherer, S., Sahto,T. ,Yamazaki, T.: On the pullback of a differential operator and its application to vector valued Eisenstein series, Comm. Math. Univ. St. Pauli 41 (1992), 1-22
[3] Harris, Michael: A note on three lemmas of Shimura. Duke Math. J. **46** (1979), no. 4, 871–879.
[4] Harris, Michael: Special values of zeta functions attached to Siegel modular forms. Ann. Sci. École Norm. Sup. (4) **14** (1981), no. 1, 77–120.
[5] Maaß, H.: Siegel's Modular Forms and Dirichlet Series (Lecture Notes in Math 216, 1971)
[6] Shimura, G.: Invariant differential operators on hermitian symmetric spaces, Ann. of Math. 132 (1990), 237-272
[7] Yamauchi, A.: These Proceedings.

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