The 42th Symposium on Transformation Groups
Abstract

Nov. 26

Definable C^r submanifolds in a definable C^r manifold

Tomohiro Kawakami (Wakayama University)

Abstract. Let X be a definable C^r manifold, Y_1, Y_2 definably compact definable C^r submanifolds of X such that $\dim Y_1 + \dim Y_2 < \dim X$ and Y_1 has a trivial normal bundle. We prove that there exists a definable isotopy $\{h_p\}_{p \in J}$ such that $h_0 = id_X$ and $h_1(Y_1) \cap Y_2 = \emptyset$.

Equivariant maps between representation spheres of cyclic p-groups

Koh Ohashi (The University of Tokyo)

Abstract. We will consider necessary conditions for the existence of equivariant maps between the unit spheres of unitary representations of a cyclic p-group G. Bartsch gave a necessary condition for some unitary representations of G by using equivariant K-theory. In this talk, we will give two necessary conditions following Bartsch's approach. One is a generalization of Bartsch's result for all unitary representations of G with fixed point free actions. The other gives a stronger necessary condition under stronger assumptions.

On the lower bounds obtained from Hom complexes

Takahiro Matsushita (The University of Tokyo)

Abstract. A coloring of a simple graph G is to assign a color to each vertex of G so that adjacent vertices have different colors. The smallest number of colores we need to color G is called the chromatic number of G, and is denoted by $\chi(G)$.

Hom complex $\mathrm{Hom}(T, G)$ is a CW-complex associated to a pair of graphs T and G. The graph T is called a homotopy test graph if the inequality

$$\chi(G) > \mathrm{conn}(\mathrm{Hom}(T, G)) + \chi(T)$$

holds for every graph G, where $\mathrm{conn}(X)$ is the largest number n such that X is n-connected. (However, we put $\mathrm{conn}(X) = -\ infty$ if $X = \emptyset$.) It is known that the complete graph K_n with $n \geq 2$ is a homotopy test graph (Lovász, Babson-Kozlov), and the odd cycle C_{2n+1} with $n \geq 1$ (Babson-Kozlov).

However, there is a graph G such that the difference between the above inequality is quite large, when $T = K_2$. In particular, Walker (1983) showed that for each positive integer n, there is a graph G such that

$$\chi(G) - \mathrm{conn}(\mathrm{Hom}(K_2, G)) > n,$$

and he also showed that there are graphs G_1, G_2 such that $\mathrm{Hom}(K_2, G_1) \simeq \mathrm{Hom}(K_2, G_2)$ but $\chi(G_1) \neq \chi(G_2)$.

In this talk we showed the following result strengthening Walker's result. Let T be a finite graph, G a graph whose chromatic number is greater than 2, and n a positive integer. Then there is a graph H which contains G as a subgraph and satisfies the following properties: the inclusion $\mathrm{Hom}(T, G) \to \mathrm{Hom}(T, H)$ is a homotopy equivalence and $\chi(H) \geq n$.

In particular, any homotopy invariant of $\mathrm{Hom}(T, G)$ does not give an upper bound for the chromatic number of G.

Nov. 27

The intersection of two real flag manifolds in a complex flag manifold

Takashi Sakai (Tokyo Metropolitan University)

Abstract. Tanaka and Tasaki studied the antipodal structure of the intersection of two real forms in Hermitian symmetric spaces of compact type. An orbit of the adjoint representation of a compact connected Lie group G admits a G-invariant Kähler structure, and called a complex flag manifold. Furthermore, any simply-connected compact homogeneous Kähler manifold is a complex flag manifold. Using a torus action, we can define (generalized) antipodal sets of a complex flag manifold. An orbit of the linear isotropy representation of the compact symmetric space G/K is called a real flag manifold, and is embedded in a complex flag manifold as a real form. In this talk, we will give a necessary and sufficient condition for two real flag manifolds, which are not necessarily congruent with each other, in a complex flag manifold to intersect transversally in terms of symmetric triads. Moreover we will show that the intersection is an orbit of a certain Weyl group and an antipodal set, if the intersection is discrete. This talk is based on a join work with Osamu Ikawa, Hiroshi Iriyeh, Takayuki Okuda and Hiroyuki Tasaki.

Totally geodesic surfaces in Riemannian symmetric spaces and nilpotent orbits

Takayuki Okuda (Hioshima University)

Abstract. Let $X = G/K$ be a Riemannian symmetric space of non-compact type with connected G. We are interested in classifications of totally geodesic (complete) submanifolds in $X = G/K$. It is known that any totally geodesic submanifold of X is homogeneous, more precisely, any such submanifold can be realized as an orbit of a reductive subgroup L of G acting on X. In this talk, we give a one-to-one correspondence between the set of G-conjugate classes of non-flat totally geodesic oriented surfaces in the Riemannian symmetric space $X = G/K$ and the set of nilpotent orbits of the real semisimple Lie algebra Lie G, which were already classified in Representation theory. This is joint work with Akira Kubo (Hiroshima Shudo Univ.), Katsuya Mashimo (Hosei Univ.) and Hiroshi Tamaru (Hiroshima Univ.).

Combination of Lorentzian transformation groups

Takayuki Masuda (Osaka University)

Abstract. We will consider classification of affine Lorentzian transformation groups acting on $(2 + 1)$-Minkowski spacetime properly discontinuously. All noncocompact affine Lorentzian transformations acting properly discontinuously are obtained by affine deformations of noncompact hyperbolic surfaces. Let a hyperbolic surface fixed. We introduce a new parameter, the affine twist parameter. Then we show that the affine deformation space can be parametrized by Margulis invariants and affine twist parameters. We will also talk about some topics associated with this theory.

A normal generating set for the Torelli group of a compact non-orientable surface

Ryoma Kobayashi (Ishikawa National College of Technology)

Abstract. The mapping class group $M(N)$ of a compact non-orientable surface N is defined as the group consisting of isotopy classes of diffeomorphisms over N fixing the boundary pointwise. The Torelli group $I(N)$ of a compact non-orientable surface N is defined as the subgroup of $M(N)$ consisting of mapping classes acting trivially on the integral first homology group of N. Hirose and the speaker have obtained a normal generating set for $I(N)$, where N is a genus $g(> 3)$ closed non-orientable surface. In this work, we obtain a normal generating set for $I(N)$, where N is a genus $g(> 3)$ compact non-orientable surface with $b(> 0)$ boundary components.

Cohomology of non-orientable toric origami manifolds

Haozhi Zeng (Osaka City University)

Abstract. Toric origami manifolds, introduced by A. Canas da Silva, V. Guillemin and A. R. Pires, are generalization of symplectic toric manifolds. In this talk we will discuss cohomology groups of some kinds of toric origami manifolds. This talk is based on the joint work with Anton Ayzenberg, Mikiya Masuda and Seonjeong Park.

Homotopy theoretical methods for rational points

Norihiko Minami (Nagoya Institute of Technology)

Abstract. In recent years, homotopy theory is applied to study rational points. In this talik, I shall survey this situation from the homotopy theoretical point of view. If possible, I would like to mention a possibility of some homotopy theoretical method for applications to rational points.

Nov. 28

Remark on the Burnside ring of A_5

Masafumi Sugimura (Okayama University)

Abstract. Let G be a finite nontrivial group and $\mathcal{F}$ a set of subgroups of G which is closed under conjugations by elements in G and under taking subgroups. Let $\mathfrak{F}$ denote the category whose objects are elements in $\mathcal{F}$ and whose morphisms are triples (H, g, K) such that H, $K \in \mathcal{F}$ and $g \in G$ with $gHg^{-1} \subset K$. We denote by $A(G)$ the Burnside ring of G. For each morphism (H, g, K), we have the associated homomorphism $(H, g, K)^* : A(K) \to A(H)$. In particular, if $H \leq K$ then $(H, e, K)^*$ agrees with $\mathrm{res}^K_H : A(K) \to A(H)$. We denote by $A(\mathfrak{F})$ the inverse limit

$$\text{inv-lim}_{\mathfrak{F}} A(\,\bullet\,) \quad \left(\subset \prod_{H \in \mathcal{F}} A(H) \right)$$

associated with the category $\mathfrak{F}$. We denote by $\mathrm{res}_{\mathcal{F}}$ the restriction homomorphism $A(G) \to A(\mathfrak{F})$ and by $A(G)|_{\mathcal{F}}$ the image of the map $\mathrm{res}_{\mathcal{F}}$. It is interesting to ask whether $A(G)|_{\mathcal{F}}$ coinsides with $A(\mathfrak{F})$. Y.Hara and M.Morimoto showed that in the case of $G = A_4$, alternating group on four letters, the answer is affirmative. We consider that issue in the case of $G = A_5$.

Realization of closed manifolds as A_5-fixed point sets

Masaharu Morimoto (Okayama University)

Abstract. Let G be the alternating group on 5 letters and let $\mathfrak{M}$ denote the family of closed smooth manifolds which can be obtained as the G-fixed point sets of smooth G-actions on disks. Let S^n denote the sphere of dimension n, let $P_{\mathbb{C}}^n$ and $P_{\mathbb{R}}^n$ denote the complex and real projective space of dimension n, respectively, and let L_m^{2n-1} denote the lens space $S(\mathbb{C}^n)/C_m$, where m is an integer ≥ 3 and

$$C_m = \{z \in \mathbb{C} \mid z^m = 1\}.$$

Let $M \in \mathfrak{M}$. We will discuss whether M can be realized as the G-fixed point sets of smooth G-actions on S^n, $P_{\mathbb{C}}^n$, $P_{\mathbb{R}}^n$ and L_m^{2n-1}.

String topology of the Borel constructions

Takahito Naito (The University of Tokyo)

Abstract. The theory of string topology is a study of algebraic structures on the homology of the free loop space (called the loop homology). It is known that the loop homology of a manifold has many algebraic structures, for example graded commutative algebra, Batalin-Vilkovisky algebra and 2-dimensional TQFT. In this talk, we will study string topology of the Borel constructions, especially TQFT structure on the loop homology. Moreover, we will introduce some properties and give some computational examples of the loop homology.

Gysin formulas for the universal Hall-Littlwood functions

Masaki Nakagawa (Okayama University)

Abstract. For certain kinds of maps (e.g., smooth maps between compact, oriented manifolds, or projections of fiber bundles), the *Gysin maps* (sometimes called *push-forwards, Umkehr maps, integration over the fiber* etc.) can be defined in ordinary cohomology. In *Schubert calculus*, there are many formulas for Gysin maps for Grassmann and flag bundles which relate *Schubert classes* with *Schur S- and P-functions* (Damon, Fulton, Harris-Tu, Pragacz). Recently Pragacz generalized the above formulas to the *Hall-Littlewood functions* which interpolate Schur S-functions and P-functions. Our main goal is to generalize the above formulas in ordinary cohomology to the *complex cobordism theory* which is *universal* among *complex-oriented generalized cohomology theories.* More precisely, we introduce the *universal* analogue of the Hall-Littlewood functions, which we call the *universal Hall-Littlwood functions*, and give analogous Gysin formulas in complex cobordism theory. This is joint work with H. Naruse.