$\boxed{1200}$ 面積ノ概念ニ基ヅク空間ノ幾何學 (續)

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前談話ニ於テハ n 次元空間ニ於ケル K 次元曲面ノ面積ガ、積分

$$O=\int L\left(x^i, \frac{\partial x^i}{\partial u^\lambda}\right) du^1 \cdots\cdots du^k$$

デ与ヘラレル空間ノ幾何學ヲ論ジタ。ココデハ面積ヲ与ヘル積分ガ最モ一般ノ形

$$O=\int F\left(x^i, \frac{\partial x^i}{\partial u^{\lambda_1}}, \cdots, \frac{\partial^m x^i}{\partial u^{\lambda_1}\cdots\partial u^{\lambda_m}}\right) du^1 \cdots du^k$$

デ与ヘラレル空間ノ幾何學化ノ問題ヲ取扱フ。F ハ勿論積分不変量ノ條件:

$$\Delta^{\lambda(s)}_{\mu} F = \sum_{(\geq)} \binom{r}{s} F_{;i}^{\lambda(s)\mu(r-s)} p^i_{\mu(r-s)\mu} = 0 \quad (s \geq 2)$$

$$\Delta^{\lambda}_{\mu} F = \sum_{r \geq 1} r F_{;i}^{\lambda\mu(r-1)} p^i_{\mu(r-1)\mu} = \delta^\lambda_\mu F$$

ヲ満足シナケレバナラナイ。

§1 $\epsilon^{\lambda(M)}_{i} = F^{-1} F_{;i}^{\lambda(M)}$、 $\epsilon^{\lambda(M)\;\mu(M)}_{i\;\;\;\;\;\;j} = F^{-1} F_{;i\;j}^{\lambda(M)\mu(M)}$

ハ i, j ニ関シ共変ベクトル, λ, μ ニ関シ反変 tensor デアル。

$$\epsilon^{\lambda_1\cdots\lambda_N\;\mu_1\cdots\mu_N}_{i_1\cdots i_{n-k};\,j_1\cdots j_{n-k}} = \sum_{\substack{(\lambda_1\cdots\lambda_M)\\(\mu_1\cdots\mu_M)}} \sum_{\substack{(i_1\cdots i_{n-k})\\(j_1\cdots j_{n-k})}}' \epsilon^{\lambda_1\cdots\lambda_M\;\mu_1\cdots\mu_M}_{(i_1\;\;\;,\;j_1)}$$

$$\cdots \epsilon^{\lambda_{N-M+1}\cdots\lambda_N\;\;\mu_{N-M+1}\cdots\mu_N}_{(i_{n-k}\;\;\;\;\;\;\;\;\;j_{n-k})} \qquad (N=(n-k)M)$$

$$u_{i_1\cdots i_{n-k}} = \varepsilon_{i_1\cdots i_{n-k}\, i_{n-k+1}\cdots i_n}\, p^{i_{n-k+1}}_{j} \cdots p^{i_n}_{k}$$

トナル量ヲツクレバ

$$\mathfrak{t}^{\lambda_1\cdots\lambda_N\ \mu_1\cdots\mu_N}_{i_1\cdots i_{n-k}\cdot j_1\cdots j_{n-k}} = u_{i_1\cdots i_{n-k}}\cdot u_{j_1\cdots j_{n-k}}\, G^{\lambda_1\cdots\lambda_N\,\mu_1\cdots\mu_N}$$

ナル如キ量 $G^{\lambda_1\cdots\lambda_N\ \mu_1\cdots\mu_N}$ ヲ求メルコトガ出来ル。ソノ行列式ガ 0 デナイトスレバ

$$g\,G^{\lambda_1\cdots\lambda_N\ \mu_1\cdots\mu_N} = g^{\lambda_1\cdots\lambda_N,\ \mu_1\cdots\mu_N} \qquad |g^{\lambda_1\cdots\lambda_N,\mu_1\cdots\mu_N}| =$$

ナル如キ座標変換デ 2荷ノ intrinsic ナ scalar g ト座標変換デ不変ナ U-tensor $g^{\lambda_1\cdots\lambda_N,\,\mu_1\cdots\mu_N}$ ヲ求メルコトガ出来ル。之ヲ用ヒテ次ノ拡張サレタ Christoffel ノ記号

$$G^{\lambda_1\cdots\lambda_N}_{\mu_1\cdots\mu_N,\nu} = \frac{1}{2} g^{\lambda_1\cdots\lambda_N,\nu_1\cdots\nu_N} \sum_{\alpha} \left(g_{\nu_1\cdots\nu_N,\mu_1\cdots\mu_N}|_{\nu} + g_{\nu_1\cdots\nu_N,\,\mu_1\cdots\mu_{\alpha-1}\nu\,\mu_{\alpha+1}\cdots}|_{\mu_\alpha} - g_{\nu_1\cdots\nu_{\alpha-1}\nu\nu_{\alpha+1}\cdots\nu_N,\,\mu_1\cdots\mu_N}|_{\nu_\alpha} \right)$$

ヲツクリ

$$G^{\lambda}_{\mu\nu} = A\, G^{\lambda_1\cdots\lambda_{N-1}\lambda}_{\lambda_1\cdots\lambda_{N-1}\,\mu,\nu} + B\,\delta^{\lambda}_{\mu}\, G^{\lambda_1\cdots\lambda_N}_{\lambda_1\cdots\lambda_N,\nu}$$

（茲ニ A, B ハ適当ナ有理数トスル）。

トオケバ，$G^{\lambda}_{\mu\nu}$ ハ V-tensor ニ対スル擬似接続ヲ定義スル。之ハ最高次ノ面素 $p\lambda^{i}_{(M+1)}$ ニ関シテ一次デアル。

今 $\mathfrak{t}^{\lambda(M)}_{i}{}^{\mu(M)}_{j}$ ガ次ノ如キ性質ヲモツト假定スル。

$$f^{k}_{\nu(M)}{}^{i}_{\lambda(M)}\, \mathfrak{t}^{\lambda(M)}_{i}{}^{\mu(M)}_{j} = Q^{k}_{j}\, \delta^{(\mu_1}_{\nu_1}\cdots\delta^{\mu_M)}_{\nu_M}$$

$$Q^{k}_{j} = \delta^{k}_{j} - P^{\nu}_{j}\, P^{k}_{\nu}$$

ナル如キ M 次ノ量 $f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j}$ ガ少クトモ一組適当ナ P^{ν}_{j} ニ対シテ存在スル。

上ノ様ナ量ガ存在スレバ，ソレハ無数ニ存在シ，且ソノ任意

ノ二ツノ間ニハ

$$F_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j} = f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j} + \varphi_{\lambda(M)}{}^{\nu}{}_{\mu(M)}{}^{j} p_{\nu}^{i} + \varphi_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{\nu} p_{\nu}^{j}$$

ナル関係ガ存在シ，逆モ亦成立スル．又 $t_{i}^{\lambda(M)}{}_{j}^{\mu(M)} \varphi_{\mu(M)}^{j} = 0$ ナル $\varphi_{\mu(M)}^{j}$ ハ必ズ $\varphi_{\mu(M)}^{\nu} p_{\nu}^{j}$ ナル形トナル，又 $\Phi_{i}^{\lambda(M)}$

$$\Psi_{i}^{\lambda(M)} \text{ ガ } \Phi_{i}^{\lambda(M)} p_{u}^{i} = \Psi_{i}^{\lambda(M)} p_{u}^{i} = 0$$

ヲ満足スル不変量ナルトキ

$$f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j} \Phi_{i}^{\lambda(M)} \Psi_{j}^{\mu(M)}$$

ハ上ノ関係ヲ満足スル $f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j}$ ニ関係シナイ *invariant* デアル．

§2 v^i ヲ任意ノ intrinsic ナ反変 vector トスレバ

$$D_{\mu}(F_{;}{}_{i}^{\lambda(M)}, v) = F^{-1}\Big[F_{;}{}_{i}^{\lambda(M)}{}_{;}{}_{j}^{\mu(M-1)\mu} v^{j}{}_{/\mu} + \big(F_{;}{}_{i}^{\lambda(M)}{}_{;}{}_{j}^{\mu(M-1)} - \delta_{(\nu_1}^{\mu_1} \cdots \delta_{\nu_{M-2}}^{\mu_{M-2}} G_{\nu_{M-2}\nu_M)}^{\mu_{M-1}} F_{;}{}_{i}^{\lambda(M)}{}_{;}{}_{j}^{\nu(M)}\big) v^{j}\Big]$$

ハ intrinsic ナ共変 vector デ μ ニ対シ U-共変 vector デアル。今

$$\nabla_{\mu} F_{;}{}_{i}^{\lambda(M)} = F_{;}{}_{i}^{\lambda(M)}{}_{/\mu} - \frac{1}{pF}\Big[t_{\mu(M-1)\mu}^{k}{}_{\nu(M)}^{k}\big(F_{jk}{}_{;}{}^{\nu(M)\mu(M-1)}{}_{i} - \delta_{(\omega_1}^{\mu_1} \cdots \delta_{\omega_{M-2}}^{\mu_{M-2}} G_{\omega_{M-1}\omega_M)}^{\mu_{M-1}} F_{;}{}_{k}^{\nu(M)}{}_{;}{}_{j}^{\omega(M)}\big) F_{;}{}_{k}^{\lambda(M)}\Big]$$

$$+ \Big(\sum_{\alpha} \delta_{\mu_1}^{\lambda_1} \cdots G_{\mu_\alpha\mu}^{\lambda_\alpha} \cdots \delta_{\mu_M}^{\lambda_M}\Big) - \delta_{\mu_1}^{(\lambda_1} \cdots \delta_{\mu_M}^{\lambda_M)} G_{\alpha\mu}^{\alpha}\Big) F_{;}{}_{i}^{\mu(M)}$$

$$(M \geqq 3)$$

$$\nabla_{\mu} F_{;}{}_{i}^{\lambda_1\lambda_2} = F_{;}{}_{i}^{\lambda_1\lambda_2}{}_{/\mu} - \frac{1}{F}\Big[t_{\mu\pi}^{k}{}_{\nu_1\nu_2}^{k}\big(F_{;}{}_{k}^{\nu_1\nu_2}{}_{;}{}_{i}^{\pi} - G_{\omega_1\omega_2}^{\pi} F_{;}{}_{k}^{\nu_1\nu_2}{}_{;}$$

$$\left.{}_{j}^{\;w_1 w_2}\right) - f_{\mu\pi}{}^{k}{}_{\nu_1\nu_2}{}^{k} E^{\pi}_{k} F;^{\nu_1\nu_2}_{i} \Big]$$

$$- G^{\alpha}_{\alpha\mu} F;^{\lambda_1\lambda_2}_{i} + 2 G^{(\lambda_1}_{\lambda\mu} F;^{\lambda_2)}_{i} \qquad (M=2)$$

茲ニ $E^{\lambda}_{i} = 2F;^{\lambda\mu}_{i}/\mu - F;^{\lambda}_{i} + \Gamma^{\lambda}_{\mu\nu} F;^{\mu\nu}_{i}$

ハ共変 vector デ且 U-vector, 又

$$p^{i}_{\nu} \nabla_{\mu} F;^{\lambda(M)}_{i} = 0$$

ヲ滿足シ, 且 $f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j}$ ノトリ方ニ無関係デアル。

又 $f_{(\lambda(M)}{}^{i}{}_{|\mu(M)}{}^{j}{}_{|}\nabla_{\lambda)} F;^{\mu(M)}_{i}$

$$= (\delta^{i}_{k} - p^{\nu}_{k} p^{i}_{\nu}) \Big[(p^{k}_{\lambda(M)\lambda} + C^{k}_{(\lambda(M)\lambda)}{}^{\mu(M+1)}_{j} p^{j}_{\mu(M+1)})$$

$$+ M^{k}_{\lambda(M)\lambda} (\chi^{i}, p^{i}_{\lambda}, \cdots, p^{i}_{\lambda(M)}) \Big]$$

ナル関係ガ成立スル。従ツテ行列

$$\left\| K^{i}_{\lambda(M+1)}{}^{\mu(M+1)}_{j} \right\| = \left\| \delta^{k}_{j} \delta^{(\mu_1}_{\lambda_1} \cdots \delta^{\mu_{M+1}}_{\lambda_{M+1}} + C^{k}_{\lambda(M+1)}{}^{\mu(M+1)}_{j} \right\|$$

ガ non-singular ダトスレバ之カラ tensor

$$T^{i}_{\lambda(M+1)} = (\delta^{i}_{j} - p^{\nu}_{j} p^{i}_{\nu})(p^{j}_{\lambda(M+1)} + H^{j}_{\lambda(M+1)})$$

ヲ次ノ関係ガ成立スル様ニ定メルコトガ出来ル。

$$f_{(\lambda(M)}{}^{i}{}_{|\mu(M)}{}^{j}{}_{|}\nabla_{\lambda)} F;^{\mu(M)}_{j} = (\delta^{i}_{j} - p^{\nu}_{j} p^{i}_{\nu}) K^{j}_{\lambda(M+1)}{}^{\mu(M+1)}_{k} T^{k}_{\mu(M+1)}$$

今 Φ ヲ任意ノ U-tensor トスレバ

$$\Phi_{\|\lambda} = \Phi_{,\lambda} - \Phi^{\lambda(M)}_{i} T^{i}_{\lambda(M)\lambda}$$

ハ経数変換デ $\Phi_{,\lambda}$ ト同ジ変換ヲスル量デ $f_{\lambda(M)}{}^{i}{}_{\mu(M)}{}^{j}$ ニ無

関係ニキマリ、且次数Mデアル。$\Phi_{\|\lambda_1\|\lambda_2\|\cdots\|\lambda_m}=\Phi_{\|\lambda_1\cdots\lambda_m}=\Phi_{\|\lambda(m)}$ナル記号ヲ導入スル。

§3 Fカラ次ノ所謂 *Synge* ノ *vector* ヲ導クコトガ出来ル。

$$F.E_i^{\lambda(S)}=\sum_{r\geqq S}(-1)^r\binom{r}{S}F_i^{((\lambda(S)\lambda(r-S))}{}_{\|\lambda(r-S)}$$

$E_i^{\lambda(S)}$ ハ径数変換デソレ等ノ間デ *linear* ニ変換サレ、径数ハ $\frac{\partial\bar{u}^\lambda}{\partial u^{\kappa_1}}$ $\frac{\partial^2\bar{u}^\lambda}{\partial u^\kappa\partial u^\beta}\cdots$ ノ *Polynomial* デアルカラ、$\Gamma_{\hat{\mu}\nu}$ 及ビソノ微分デ *intrinsic* ニスル事ガ出来ル。但シ $\Gamma_{\hat{\mu}\nu}$ ハ $G_{\hat{\mu}\nu}$ ノ $M+1$ 次ノ面素ヲ $-(\delta_j^i-P_j^\nu p_\nu^i)H_{\lambda}^{j}{}_{(M+1)}$ デオキカヘタモノトスル。之ヲ $\ell_i^{\lambda(S)}$ トスル。

$$\ell_i^{\lambda(S)}p_\nu^i=0\qquad(S\geqq 2)$$

$$\ell_{\hat{i}}^{\hat{\lambda}}p_{\hat{\nu}}^{i}=\delta_{\hat{\nu}}^{\hat{\lambda}}$$

今 $\ell_{\lambda}{}^{i}{}_{(M)}u_{(M)}^{j}=(\delta_k^i-\ell_k^\nu p_\nu^i)(\delta_k^j\ell_k^\nu p_\nu^j)f_{\lambda}{}^{k}{}_{(M)}u_{(M)}^{k}$

トオケバ之ハ $f_{\lambda}{}^{i}{}_{(M)}u_{(M)}^{j}$ ニ無関係デアル。

$$\Lambda_{j\mu}^{i}=\frac{1}{pF}\ell_{\mu(M-1)\mu\lambda(M)}^{i\qquad\qquad k}\Big(F_{;}{}^{\lambda(M)}_{k}{}_{;}{}^{\mu(M-1)}_{j}$$

$$-\sum\delta_{(\omega_1}^{\mu_1}\Gamma_{\omega_{M-1}\omega_M)}^{\mu_{M-1}}F_{;}{}^{\lambda(M)}_{k}{}_{;}{}^{\mu(M)}_{j}\Big)$$

ナル量ヲ用ヒ任意ノ反変 *vector* v^i ヨリ

$$(\delta_j^i-\ell_j^\nu p_\nu^i)v^j{}_{|\mu}+\Lambda_{j\mu}^{i}v^j$$

ナル *invariant* ヲ導クコトガ出来ル。之ト

$$(\ell_j^\nu v^j)_{;\mu}=\ell_j^\nu v^j{}_{|\mu}+p_\nu^i(\ell_{j\|\lambda}^{i}+\Gamma_{\mu\lambda}^{\nu}\ell_j^{\mu})$$

ナル v-*vector* ヨリ

$$v^i{}_{;\lambda}=\frac{\partial v^i}{\partial u^\lambda}+\Gamma_{j\lambda}^{i}v^j,$$

$$\Gamma^{i}_{j\lambda} = \Lambda^{i}_{j\lambda} + p^{i}_{\nu}\left(\ell^{\nu}_{j\mu\lambda} + \Gamma^{\nu}_{\mu\lambda} \ell^{\mu}_{j} \right)$$

ナル曲面ニ沿フ共変微分ガ定義サレル.

$$\sum_{r\geq 0} (r+1)\,\Gamma^{i}_{j\lambda\,j\,k}{}^{(\mu(r)\lambda)}\,dp^{k}_{i(r)}$$

ヲ補正シテ共変微分

$$\delta v^{i} = dv^{i} + \sum_{r=0}^{M-1} \Gamma^{i\ \mu(r)}_{j\ k}\,dp^{k}_{\mu(r)}\,v^{j}$$

ヲ得ル。

$$\ell^{\ k\ \ \ i}_{\mu(M)\lambda(M)}\,\delta F_{j}{}_{i}^{\lambda(M)}$$

$$\ell^{\ k\ \ \ i}_{\mu(M)\lambda(M)} \sum_{1\geq s} \binom{r}{s} F_{j}{}_{i}^{\lambda(M)}{}_{j\,j}^{\mu(r-s),\mu(s)}\,dp^{j}_{\mu(r-s)}$$

ヲ補正シテ基接続

$$\begin{cases} \delta p^{i}_{\lambda(M)} = \left(\delta^{i}_{j} - \ell^{\nu}_{j} p^{i}_{\nu} \right) dp^{j}_{\lambda(M)} + \sum_{r=0}^{M-1} \Lambda^{i\ \ \mu(r)}_{\lambda(M)\,j}\,dp^{j}_{\mu(r)} \\ --- \\ \delta p^{i}_{\lambda} = \left(\delta^{i}_{j} - \ell^{\nu}_{j} p^{i}_{\nu} \right) dp^{j}_{\lambda} + \Lambda^{i}_{j\lambda}\,dx^{j} \end{cases}$$

U-tensor ノ共変微分ハ

$$\delta v^{\lambda} = p^{\lambda}_{i}\,\delta\left(p^{i}_{\mu} v^{\mu} \right)$$

即チ $\delta v^{\lambda} = dv^{\lambda} + p^{\lambda}_{i}\left(dp^{i}_{\mu} + dp^{j}_{\mu} \sum_{r=0}^{M-1} \Gamma^{i\ \nu(r)}_{j\ k}\,dp^{k}_{\nu(r)} \right) v^{\mu}$

ニヨリ定義スレバヨイ。

§4.

今 $M \geq 3$ トシ $a_{\alpha} = \ell^{i\ \ \ \ \ \ \ j}_{\alpha\beta(M-1)\,\gamma(M)}\,\ell_{i}^{\beta(M-1)}\,\ell_{j}^{\gamma(M)}$

ナル U-vector ヲ考ヘル. $M=2$ ナラバ $\ell_{i}^{\beta(M-1)}$ ノ代リニ $\ell = \ell^{i\ \ \ \ \ \ j}_{\lambda(M)\mu(M)}\,\ell_{i}^{\lambda(M)}\,\ell_{j}^{\mu(M)}$

ヲ用ヒテツクツタ Synge ノ M−1−Vector ヲ用ヒテ U-vector a_{α} ヲ導イテオク,

$$a = g^{\lambda_1\cdots\lambda_N\cdot\mu_1\cdots\mu_N}\,a_{\lambda}\cdots a_{\lambda_N} a_{\mu}\cdots a_{\mu_N}$$

$$\alpha^{\lambda}=g^{\lambda\lambda_2\cdots\lambda_N,\,\mu_1\cdots\mu_N}\,a_{\lambda_2}\cdots a_{\lambda_N}a_{\mu_1}\cdots a_{\mu_N}$$

$$\alpha^{\lambda\mu}=g^{\lambda\lambda_2\cdots\lambda_N,\,\mu\mu_2\cdots\mu_N}\,a_{\lambda_2}\cdots a_{\lambda_N}a_{\mu_2}\cdots a_{\mu_N}$$

トオク。

今 $g^{\lambda\mu}=p\,(Na\cdot a^{\lambda\mu}-(N-1)\,a^{\lambda}a^{\mu})$

ノ行列式ガ 0 デナイトシ

$$\pm|g^{\lambda\mu}|=L^{-2}$$

ニヨリ p ヲ定メル。次ニ

$$g''_{ij}=\sigma\cdot g_{\lambda_1\mu_1}\cdots g_{\lambda_M\mu_M}\,\ell_i^{\lambda(M)}\,\ell_j^{\mu(M)}$$

トオキ $\pm g''_{[i_1|j_1|}\cdots g''_{i_{n-k}]j_{n-k}}=v_{i_1\cdots i_{n-k}}v_{j_1\cdots j_{n-k}}$

ニヨリ σ ヲ定メル。然ルトキハ

$$g_{ij}=g''_{ij}+\ell_i^{\nu}\ell_j^{\lambda}g_{\nu\lambda}$$

ニヨリ基本 tensor g_{ij} ガ定義サレ

$g_{\lambda\mu}=p_{\lambda}^{i}p_{\mu}^{j}g_{ij}$, $\ell_i^{\lambda}=g_{ij}\,g^{\lambda\mu}p_{\mu}^{j}$ ガ成立スル。

之ヲ用ヒテ Euclid 接続ヲ導入スルコトガ出来ル。

$$\delta v^{i}=dv^{i}+\omega^{i}_{j}(d)\,v^{j}$$

$$\omega^{i}_{j}(d)=\sum_{\gamma=0}^{M}\gamma^{i\,\lambda(\gamma)}_{j\,k}\,dp^{k}_{\lambda(\gamma)}$$

$$\gamma^{i\,\lambda(\gamma)}_{j\,k}=\frac{1}{2}g^{ih}\,g^{*}_{hj;k}{}^{\lambda(\gamma)}\qquad(\gamma\geqq 1)$$

$$\gamma^{i}_{jk}=\frac{1}{2}g^{ih}\{g^{*}_{hj,k}+g^{*}_{hk,j}-g^{*}_{jk,h}\}$$

然ニ

$$dg_{ij}=\sum_{\gamma=0}^{M}g^{*}_{ij,k}{}^{\lambda(\gamma)}\,\delta p^{k}_{\lambda(\gamma)}$$

トスル。