1103
~~251~~ 3級パスノ空間∇_nノ中ノ
∇_{n-1}ニ就テ

朝長 康郎(東大學生)

§0. 3級パスノ理論ハ数年前ニ本部均先生ガ研究サレタ。[1] ソコデ今蛇足ナガラソノ超曲面ノ理論ヲ考ヘテ見ヨウ。

先ヅ、本部先生ニ随ツテ、射影的媒変数ヲ持ツ3級パスノ空間∇_nノ構造ヲ決メル。基本的ナ量ノコトヲ簡単ニ説明スル。

(0.1) $T^{\lambda}=x^{(3)\lambda}+H^{\lambda}(x, x^{(1)}, x^{(2)})=0$

デ定義サレタ3級パスが、射影的媒変数ヲ持ツト云フナラバH^{λ}ハ次ノ條件ヲ満足シナケレバナラヌ。

(0.2) a) $H^{\lambda}_{(1)\nu}x^{(1)\nu}+2H^{\lambda}_{(2)\nu}x^{(2)\nu}=3H^{\lambda}$

b) $H^{\lambda}_{(2)\nu}x^{(1)\nu}=-3x^{(2)\lambda}$

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(1) H. Idomba. Projektive Transformation eines Systems der gewöhnlichen Differential=gleichungen dritter Ordnung.
帝國學士院記事 Vol. 13.

H. Idomba. Die projektive Theorie der "Paths" 3-ter Ordnung $x^{(3)i}+H^{i}(x,x^{(1)},x^{(2)})=0$
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∇_n ノ基接線トシテ、次ノ二ツヲ採ル。

(0.3) a) $\delta x^{(1)\lambda} = dx^{(1)\lambda} + \frac{1}{3} H^{\lambda}_{(2)\nu} dx^{\nu}$

b) $\delta x^{(2)\lambda} = dx^{(2)\lambda} + \frac{2}{3} H^{\lambda}_{(2)\nu} dx^{(1)\nu} + \frac{1}{3} H^{\lambda}_{(1)\nu} dx^{\nu}$

普通ノ曲線ニ沿ツテハ $\frac{\delta x^{(1)\lambda}}{dt} = 0$，パスニ沿ツテハ $\frac{\delta x^{(2)\lambda}}{dt} = 0$ デアル。

∇_n ノヴェクトルノ共変微分ハ

(0.4) $\delta v^{\lambda} = dv^{\lambda} + \omega^{\lambda}_{\mu} v^{\mu}$.

茲ニ $\omega^{\lambda}_{\mu} = \underset{(0)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu} dx^{\nu} + \underset{(1)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu} \delta x^{(1)\nu}$

(0.5) a) $\underset{(0)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu} = \frac{1}{3} H^{\lambda}_{(2)\mu(1)\nu} - \frac{2}{9} H^{\lambda}_{(2)\mu(2)\sigma} H^{\sigma}_{(2)\nu}$

b) $\underset{(1)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu} = \frac{2}{3} H^{\lambda}_{(2)\mu(2)\nu}$

デアタヘル。

(0.4) ハ又次ノ様ニモ書ケル。

$$\delta v^{\lambda} = \overset{(0)}{\nabla}_{\nu} v^{\lambda} \cdot dx^{\nu} + \overset{(1)}{\nabla}_{\nu} v^{\lambda} \cdot \delta x^{(1)\nu} + \overset{(2)}{\nabla}_{\nu} v^{\lambda} \delta x^{(2)\nu}$$

茲ニ

(0.6) $\overset{(0)}{\nabla}_{\nu} v^{\lambda} = \overset{(0)}{\overline{\nabla}}_{\nu} v^{\lambda} + \underset{(0)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu} v^{\mu}$, $\overset{(1)}{\nabla}_{\nu} v^{\lambda} = \overset{(1)}{\overline{\nabla}}_{\nu} v^{\lambda} + \underset{(0)}{\overset{k}{\Gamma}}{}^{\lambda}_{\mu\nu} v^{\mu}$

$\overset{(2)}{\nabla}_{\nu} v^{\lambda} = \overset{(2)}{\overline{\nabla}}_{\nu} v^{\lambda}$

$$(0.7)\quad a)\quad \overline{\nabla}^{(0)}_{\nu} = \frac{\partial}{\partial x^{\nu}} - \frac{1}{3} H^{\mu}_{(2)\nu} \frac{\partial}{\partial x^{(1)\mu}} - \left(\frac{1}{3} H^{\mu}_{(1)\nu} - \frac{2}{9} H^{\mu}_{(2)\sigma} H^{\sigma}_{(2)\nu} \right) \frac{\partial}{\partial x^{(2)\mu}}$$

$$b)\quad \overline{\nabla}^{(1)}_{\nu} = \frac{\partial}{\partial x^{(1)\nu}} - \frac{2}{3} H^{\mu}_{(2)\nu} \frac{\partial}{\partial x^{(2)\mu}}$$

$$c)\quad \overline{\nabla}^{(2)}_{\nu} = \frac{\partial}{\partial x^{(2)\nu}}$$

∇_κ ノ曲率テンソル[1]ハ v^λ ヲ任意ノヴェクトルトスレバ

$$(0.8)\quad \underset{2}{\delta}\underset{1}{d} v^{\lambda} - \underset{1}{d}\underset{2}{\delta} v^{\lambda} = v^{\mu}\Big[\Big(R^{\lambda(0)(0)}_{\mu\sigma\pi} + \overset{*}{\underset{(1)}{\Gamma}}{}^{\lambda}_{\mu\tau} S^{(0)(0)\tau}_{\sigma\pi(1)}\Big)\underset{1}{d}x^{\sigma}\underset{2}{d}x^{\pi} + R^{\lambda(1)(1)}_{\mu\sigma\pi}\underset{1}{\delta}x^{(1)\sigma}\underset{2}{\delta}x^{(1)\pi} + R^{\lambda(0)(1)}_{\mu\sigma\pi}\Big[\underset{1}{d}x^{\sigma}\underset{2}{\delta}x^{(1)\pi}\Big] + R^{\lambda(0)(2)}_{\mu\sigma\pi}\Big[\underset{1}{d}x^{\sigma}\underset{2}{\delta}x^{(2)\pi}\Big] + R^{\lambda(1)(2)}_{\mu\sigma\pi}\Big[\underset{1}{\delta}x^{(1)\sigma}\underset{2}{\delta}x^{(2)\pi}\Big]\Big]$$

ノ如キ意味ヲ有シ、ソノ形ハ次ノ通リデアル。

$$(0.9)\quad a)\quad R^{\lambda(0)(0)}_{\mu\sigma\pi} = \overline{\nabla}^{(0)}_{\pi} \overset{\vee}{\underset{(0)}{\Gamma}}{}^{\lambda}_{\mu\sigma} - \overline{\nabla}^{(0)}_{\sigma} \overset{\vee}{\underset{(0)}{\Gamma}}{}^{\lambda}_{\mu\pi} + \overset{\vee}{\underset{(0)}{\Gamma}}{}^{\tau}_{\mu\sigma} \overset{\vee}{\underset{(0)}{\Gamma}}{}^{\lambda}_{\tau\pi} - \overset{\vee}{\underset{(0)}{\Gamma}}{}^{\tau}_{\mu\pi} \overset{\vee}{\underset{(0)}{\Gamma}}{}^{\lambda}_{\tau\sigma}$$

$$b)\quad R^{\lambda(1)(1)}_{\mu\sigma\pi} = \overline{\nabla}^{(1)}_{\pi} \overset{\vee}{\underset{(1)}{\Gamma}}{}^{\lambda}_{\mu\sigma} - \overline{\nabla}^{(1)}_{\sigma} \overset{\vee}{\underset{(1)}{\Gamma}}{}^{\lambda}_{\mu\pi} + \overset{\vee}{\underset{(1)}{\Gamma}}{}^{\tau}_{\mu\sigma} \overset{\vee}{\underset{(1)}{\Gamma}}{}^{\lambda}_{\tau\pi} - \overset{\vee}{\underset{(1)}{\Gamma}}{}^{\tau}_{\mu\pi} \overset{\vee}{\underset{(1)}{\Gamma}}{}^{\lambda}_{\tau\sigma}$$

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(1) 曲率テンソルハ全然筆者ガ附加ヘタモノデアル。

c) $$R^{\lambda(0)(1)}_{\mu\sigma\pi} = \overset{(1)}{\overline{\nabla}}_{\pi} \underset{(0)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\mu\sigma} - \overset{(0)}{\overline{\nabla}}_{\sigma} \underset{(1)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\mu\pi} - \underset{(0)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\varepsilon\sigma} \underset{(1)}{\overset{\nu}{\Gamma}}{}^{\varepsilon}_{\mu\pi}$$

$$+ \underset{(1)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\varepsilon\pi} \underset{(0)}{\overset{\nu}{\Gamma}}{}^{\varepsilon}_{\mu\sigma} + \underset{(1)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\mu\varepsilon} \underset{(0)}{\overset{\nu}{\Gamma}}{}^{\varepsilon}_{\sigma\pi}$$

d) $$R^{\lambda(0)(2)}_{\mu\sigma\pi} = \overset{(2)}{\overline{\nabla}}_{\pi} \underset{(0)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\mu\sigma} + \frac{1}{3} \underset{(1)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\mu\tau} \overset{(2)}{\overline{\nabla}}_{\pi} H^{\tau}_{(2)\sigma}$$

e) $$R^{\lambda(1)(2)}_{\mu\sigma\pi} = \overset{(2)}{\overline{\nabla}}_{\pi} \underset{(1)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\mu\sigma}$$

又捩率テンソル[2]ハ次ノヤウナ意味ヲ持ツ。

(0.10)

a) $$\underset{2}{\delta}\underset{1}{d}x^{\lambda} - \underset{1}{\delta}\underset{2}{d}x^{\lambda} = S^{(0)(0)\lambda}_{\mu\sigma(0)} \underset{1}{d}x^{\mu} \underset{2}{d}x^{\sigma}$$

$$+ S^{(0)(1)\lambda}_{\mu\sigma(0)} \left[\underset{1}{d}x^{\mu} \underset{2}{\delta}x^{(1)\sigma} \right]$$

b) $$\underset{2}{\delta}\underset{1}{\delta}x^{(1)\lambda} - \underset{1}{\delta}\underset{2}{\delta}x^{(1)\lambda} = S^{(0)(0)\lambda}_{\mu\sigma(1)} \underset{1}{d}x^{\mu} \underset{2}{d}x^{\sigma}$$

$$+ S^{(0)(2)\lambda}_{\mu\sigma(1)} \left[\underset{1}{d}x^{\mu} \underset{2}{\delta}x^{(2)\sigma} \right] + S^{(0)(1)\lambda}_{\mu\sigma(1)} \left[\underset{1}{d}x^{\mu} \underset{2}{\delta}x^{(1)\sigma} \right]$$

c) $$\underset{2}{\delta}\underset{1}{\delta}x^{(2)\lambda} - \underset{1}{\delta}\underset{2}{\delta}x^{(2)\lambda} = S^{(0)(0)\lambda}_{\mu\sigma(2)} \underset{1}{d}x^{\mu} \underset{2}{d}x^{\sigma}$$

$$+ S^{(1)(1)\lambda}_{\mu\sigma(2)} \underset{1}{\delta}x^{(1)\mu} \underset{2}{\delta}x^{(1)\sigma} + S^{(0)(1)\lambda}_{\mu\sigma(2)} \left[\underset{1}{d}x^{\mu} \underset{2}{\delta}x^{(1)\sigma} \right]$$

a) $$S^{(0)(0)\lambda}_{\mu\sigma(0)} = \underset{(0)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\mu\sigma} - \underset{(0)}{\overset{\nu}{\Gamma}}{}^{\lambda}_{\sigma\mu}$$

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(2) 捩率テンソルハ会部、本部先生ノアタヘラレタモノノ2倍トシテアル。

b) $$S^{(0)(1)\lambda}_{\mu\sigma(0)} = \overset{k}{\underset{(1)}{\Gamma}}{}^{\lambda}_{\mu\sigma}$$

c) $$S^{(0)(0)\lambda}_{\mu\sigma(1)} = \frac{1}{3}\overline{\nabla}^{(0)}_{\sigma} H^{\lambda}_{(2)\mu} - \frac{1}{3}\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(2)\sigma}$$

d) $$S^{(0)(2)\lambda}_{\mu\sigma(1)} = \frac{1}{2}\overset{k}{\underset{(1)}{\Gamma}}{}^{\lambda}_{\mu\sigma}$$

e) $$S^{(0)(0)\lambda}_{\mu\sigma(2)} = \frac{1}{3}\overline{\nabla}^{(0)}_{\sigma} H^{\lambda}_{(1)\mu} - \frac{2}{9}\left(\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(2)\pi}\right)\cdot H^{\pi}_{(2)\sigma}$$

$$- \frac{1}{3}\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(1)\sigma} + \frac{2}{9}\left(\overline{\nabla}^{(2)}_{\sigma} H^{\lambda}_{(2)\pi}\right) H^{\pi}_{(2)\mu}$$

f) $$S^{(1)(1)\lambda}_{\mu\sigma(2)} = S^{(0)(0)\lambda}_{\mu\sigma(0)}$$

g) $$S^{(0)(1)\lambda}_{\mu\sigma(2)} = \frac{1}{3}\overline{\nabla}^{(1)}_{\sigma} H^{\lambda}_{(1)\mu} - \frac{2}{3}\overline{\nabla}^{(0)}_{\mu} H^{\lambda}_{(2)\sigma} - \frac{2}{9} H^{\tau}_{(2)\mu}\overline{\nabla}^{(1)}_{\sigma} H^{\lambda}_{(2)\tau}$$

h) $$S^{(0)(1)\lambda}_{\mu\sigma(1)} = S^{(0)(0)\lambda}_{\mu\sigma(0)}$$

§1. V_n ノ中ノ超曲面 V_{n-1} ノ方程式ヲ

$$x^{\lambda} = x^{\lambda}(y, y^{\dot{1}}, y^{\dot{2}}, \dots\dots, y^{n-1})$$

トスルトキ、此ノ V_{n-1} ノ上ノ與ヘラレタ3級パス（矢張リ射影的媒介數ヲ持ツ）

$$T^{i} = y^{(3)i} + H^{i}(y, y^{(1)}, y^{(2)}) = 0$$

$$(i = \dot{1}, \dot{2}, \dots\dots, n-1)$$

ノ H^{i} カラ全ク V_n ノトキト全様ニ諸種ノ量ヲ決メル。

但シ，ギリシヤ字ハ V_n ニ關スル index ニ，ラテン字ハ V_{n-1} ニ關スル index ニ用ヒルコトヽスル。今

$$(1.1)\quad \mathfrak{J}^\lambda = H^\lambda - \xi^\lambda_i H^i + 3\xi^\lambda_{jk} y^{(2)j} y^{(1)k} + \xi^\lambda_{jkh} y^{(1)j} y^{(1)k} y^{(1)h}$$

ト置クト

$$\left(\text{茲ニ}\ \xi^\lambda_i = \frac{\partial x^\lambda}{\partial y^i},\quad \xi^\lambda_{jk} = \frac{\partial^2 x^\lambda}{\partial y^j \partial y^k},\quad \xi^\lambda_{jkh} = \frac{\partial^3 x^\lambda}{\partial y^j \partial y^k \partial y^h}\right)$$

$\mathfrak{J}^\lambda$ ハ V_n ノヴエクトルデ V_{n-1} ノ scalar トナル。假定ニヨリ

$$(1.2)\quad a)\ \mathfrak{J}^\lambda_{(1)s} y^{(1)s} + 2\mathfrak{J}^\lambda_{(2)s} y^{(2)s} = 3\mathfrak{J}^\lambda$$

$$b)\ \mathfrak{J}^\lambda_{(2)s} y^{(1)s} = 0$$

デナケレバナラナイ，(1.1)カラ得ラレル。

$$(1.3)\quad a)\ \mathfrak{J}^\lambda_{(1)s} = H^\lambda_{(1)\nu}\xi^\nu_{is} + 2H^\lambda_{(2)\nu}\xi^\nu_{is} y^{(1)i} - \xi^\lambda_i H^i_{(1)s} + 3\xi^\lambda_{is} y^{(2)i} + 3\xi^\lambda_{sjk} y^{(1)j} y^{(1)k}$$

$$b)\ \mathfrak{J}^\lambda_{(2)s} = H^\lambda_{(2)\nu}\xi^\nu_{is} - \xi^\lambda_i H^i_{(2)s} + 3\xi^\lambda_{is} y^{(1)i}$$

ヲ利用スレバ V_n ノ垂直ニ對シテ

(1.4) a) $$\overline{\nabla}^{0}_{l}\Phi = \xi^{\lambda}_{l}\overline{\nabla}^{(0)}_{\lambda}\Phi + \frac{1}{3}J^{\lambda}_{(2)l}\overline{\nabla}^{(1)}_{\lambda}\Phi + \frac{1}{3}\left(\overline{\nabla}^{(1)}_{l}J^{\lambda}\right)\overline{\nabla}^{(2)}_{\lambda}\Phi$$

b) $$\overline{\nabla}^{(1)}_{l}\Phi = \xi^{\lambda}_{l}\overline{\nabla}^{(1)}_{\lambda}\Phi + \frac{2}{3}J^{\lambda}_{(2)l}\overline{\nabla}^{(2)}_{\lambda}\Phi$$

c) $$\overline{\nabla}^{(2)}_{l}\Phi = \xi^{\lambda}_{l}\overline{\nabla}^{(2)}_{\lambda}\Phi$$

ノ如キ関係が成ル。

又 ∇_{n-1} ニ属スル $x^{(1)\lambda}$, $x^{(2)\lambda}$ 即チ

$$x^{(1)\lambda} = \xi^{\lambda}_{i}y^{(1)i}, \quad x^{(2)\lambda} = \xi^{\lambda}_{i}y^{(2)i} + \xi^{\lambda}_{ij}y^{(1)i}y^{(1)j}$$

ヲ用ヒタトキハ

(1.5) a) $$\delta x^{(1)\lambda} = \xi^{\lambda}_{i}\delta y^{(1)i} + \frac{1}{3}J^{\lambda}_{(2)i}dy^{i}$$

b) $$\delta x^{(2)\lambda} = \xi^{\lambda}_{i}\delta y^{(2)i} + \frac{2}{3}J^{\lambda}_{(2)i}\delta y^{(1)i} + \frac{1}{3}\left(\overline{\nabla}^{(1)}_{j}J^{\lambda}\right)dy^{j}$$

が成立スルカラ、$\delta\xi^{\lambda}_{j}$ ヲ計算スレバ

(1.6) $$\delta\xi^{\lambda}_{j} = F^{\lambda}_{jk}dy^{k} + \frac{2}{3}J^{\lambda}_{(2)j(2)k}\delta y^{(1)k}$$

(1.7) $$F^{\lambda}_{jk} = \xi^{\lambda}_{jk} + \underset{(0)}{\Gamma}{}^{\lambda}_{\mu\nu}\xi^{\mu}_{j}\xi^{\nu}_{.k} - \xi^{\lambda}_{i}\underset{(0)}{\Gamma}{}^{i}_{jk} + \frac{1}{3}\underset{(1)}{\Gamma}{}^{\lambda}_{\mu\nu}\xi^{\nu}_{.j}J^{\nu}_{(2)k}$$

(1.8) $$\frac{2}{3}J^{\lambda}_{(2)j(2)k} = \underset{(1)}{\Gamma}{}^{\lambda}_{\mu\nu}\xi^{\mu}_{j}\xi^{\nu}_{,k} - \xi^{\lambda}_{i}\underset{(1)}{\Gamma}{}^{i}_{jk}$$

ノ如キ関係ガ生ズル。

(1.3)ト(1.4)カラ

$$(1.9)\quad \Gamma^{\lambda}_{jk}=\frac{1}{3}\overset{(1)}{\nabla}_{k}\mathfrak{T}^{\lambda}_{(2)j}$$

トナルコトガ直ゲ分ル。(1.6)ヨリ

V_{n-1} ニ切スルヴェクトル $v^{\lambda}=\xi^{\lambda}_{i}v^{i}$ ヲ V_{n-1} ニ沿ッテ共変微分スレバ

$$(1.10)\quad \delta v^{\lambda}=\xi^{\lambda}_{i}\delta v^{i}+v^{j}\Big(\Gamma^{\lambda}_{jk}dy^{k}$$
$$+\frac{2}{3}\mathfrak{T}^{\lambda}_{(1)j(2)k}\delta y^{(1)k}\Big)$$

コヽデ、到ル所 $\mathfrak{T}^{\lambda}=0$ ノ如キ V_{n-1} ヲ *totally geodesic*-V_{n-1} ト名付ケルナラバ(1.1)ヲ書キ直シテ得ラレル。

$$\mathfrak{T}^{\lambda}=T^{\lambda}-\xi^{\lambda}_{i}T^{i}$$

ヨリ, *totally geodesic* V_{n-1} デハ、V_{n-1} ノパスハ必ズ V_{n} ノパストナッテキルコトガ分ル。次ニ到ル所 $\mathfrak{T}^{\lambda}_{(2)k}=0$ ノ如キ V_{n-1} ヲ *semi-geodesic* V_{n-1} ト呼ブコトニスレバ、(1.9)(1.2)ヨリ コノ V_{n-1} ハ次ノ性質ヲ有シ。

a) $\delta\xi^{\lambda}_{j}=0$

b) $\mathfrak{T}^{\lambda}$ ガ $x^{(1)i}$ ノ三次ノ全次式

即チ、*semi-geodesic* V_{n-1} ハ、ソレニ切スルヴェクトル v^{λ} ヲ V_{n} ノ意味デ V_{n-1} ニ沿ッテ平行ニ移動スレバ

$\underline{\nabla_{n-1}\text{デモ平行ニ移動スル。}}$ト云フ特徴ヲ持ツ。モトニ帰

ッテ

(1.7) (1.8) ヨリ直チニ

$$(1.11)\quad \xi^{\mu}_{j} w^{\lambda}_{\mu} = \xi^{\lambda}_{i} w^{i}_{j} - \xi^{\lambda}_{jk} dy^{k} + F^{\lambda}_{jk} dy^{k} + \frac{2}{3} J^{\lambda}_{(2)j(2)k} \delta y^{(1)k}$$

ヲ得ル。

§2. (1.11) ノ積分可能條件ヲ求メルニハ (1.6)(1.10)

ヲ利用シテ

$$v^{\lambda} = \xi^{\lambda}_{j} v^{j}$$

ナル任意ノ v^{λ} ニ對シテ $\underset{2}{\delta}\underset{1}{\delta} v^{\lambda} - \underset{1}{\delta}\underset{2}{\delta} v^{\lambda}$ ヲ計算シタ方ガヨ

イ。即チ出テ来ル條件ハ次ノ如シ。

(2,1)

$$\begin{aligned}
a)\quad & \overset{\vee}{R}{}^{\lambda(0)(0)}_{\mu\sigma\pi} \xi^{\mu}_{.j} \xi^{\sigma}_{.k} \xi^{\pi}_{h} + \frac{1}{9} R^{\lambda(1)(1)}_{\mu\sigma\pi} J^{\sigma}_{(2)k} J^{\pi}_{(2)h} \xi^{\mu}_{j} \\
& + \frac{1}{3} R^{\lambda(0)(1)}_{\mu\sigma\pi} \xi^{\mu}_{j} \left(\xi^{\sigma}_{.k} J^{\pi}_{(2)h} \xi^{\sigma}_{,h} J^{\pi}_{(2)k} \right) \\
& + \frac{1}{3} R^{\lambda(0)(2)}_{\mu\sigma\pi} \xi^{\mu}_{j} \left[\xi^{\sigma}_{k} \left(\overline{\nabla}^{(1)}_{h} J^{\pi} \right) - \xi^{\sigma}_{h} \left(\overline{\nabla}^{(1)}_{k} J^{\pi} \right) \right] \\
& + \frac{1}{9} R^{\lambda(1)(2)}_{\mu\sigma\pi} \xi^{\mu}_{j} \left[J^{\sigma}_{(2)k} \left(\overline{\nabla}^{(1)}_{h} J^{\pi} \right) - J^{\sigma}_{(2)h} \left(\overline{\nabla}^{(1)}_{k} J^{\pi} \right) \right] \\
& = \xi^{\lambda}_{i} \overset{\vee}{R}{}^{i(0)(0)}_{jkh} + \nabla^{(0)}_{h} F^{\lambda}_{jk}{}^{(1)} - \nabla^{(0)}_{k} F^{\lambda}_{jh} \\
& + F^{\lambda}_{ja} S^{(0)(0)a}_{kh(0)} + \frac{2}{3} J^{\lambda}_{(2)j(2)a} S^{(0)(0)a}_{kh(1)}
\end{aligned}$$

脚註(1) 次頁へ

茲 = $\overset{\lambda}{R}{}^{\lambda(0)(0)}_{\mu\sigma\pi} = R^{\lambda(0)(0)}_{\mu\sigma\pi} + \underset{(1)}{\Gamma}{}^{\lambda}_{\mu\tau} S^{(0)(0)\tau}_{\sigma\pi(1)}$

b) $R^{\lambda(1)(1)}_{\mu\sigma\pi} \xi^{\mu}_{,j} \xi^{\sigma}_{k} \xi^{\pi}_{h}$

$+ \frac{2}{3}\left(R^{\lambda(1)(2)}_{\mu\sigma\pi} \xi^{\sigma}_{k} J^{\pi}_{(2)h} - R^{\lambda(1)(2)}_{\mu\sigma\pi} \xi^{\sigma}_{h} J^{\pi}_{(2)k}\right) \xi^{\mu}_{,j}$

$= \xi^{\lambda}_{i} R^{i(1)(1)}_{j\,k\,h} + \frac{2}{3}\left(\nabla^{(1)}_{h} J^{\lambda}_{(2)j(2)k} - \nabla^{(1)}_{k} J^{\lambda}_{(2)j(2)h}\right)$

c) $R^{\lambda(0)(1)}_{\mu\sigma\pi} \xi^{\mu}_{j} \xi^{\sigma}_{k} \xi^{\pi}_{h} + \frac{1}{3} R^{\lambda(1)(1)}_{\mu\sigma\pi} \xi^{\mu}_{j} J^{\sigma}_{(2)k} \xi^{\pi}_{h}$

$+ \frac{2}{3} R^{\lambda(0)(2)}_{\mu\sigma\pi} \xi^{\mu}_{,j} \xi^{\sigma}_{k} J^{\pi}_{(2)h}$

$- \frac{1}{3} R^{\lambda(1)(2)}_{\mu\sigma\pi} \xi^{\mu}_{j} \xi^{\sigma}_{h} \left(\overline{\nabla}^{(1)}_{k} J^{\pi}_{\cdot}\right)$

$+ \frac{2}{9} R^{\lambda(1)(2)}_{\mu\sigma\pi} J^{\sigma}_{(1)k} J^{\pi}_{(2)h} \xi^{\mu}_{j}$

$= \xi^{\lambda}_{i} R^{i(0)(1)}_{j\,k\,h} + \nabla^{(1)}_{h} F^{\lambda}_{jk} - \frac{2}{3} \nabla^{(0)}_{k} J^{\lambda}_{(2)j(2)h}$

$+ F^{\lambda}_{ja} S^{(0)(1)a}_{k\,h(0)} + \frac{2}{3} J^{\lambda}_{(2)j(2)a} S^{(0)(1)a}_{k\,h(1)}$

d) $R^{\lambda(0)(2)}_{\mu\sigma\pi} \xi^{\mu}_{,j} \xi^{\sigma}_{k} \xi^{\pi}_{h} + \frac{1}{3} R^{\lambda(1)(2)}_{\mu\sigma\pi} \xi^{\mu}_{j} J^{\sigma}_{(2)k} \xi^{\pi}_{h}$

$= \xi^{\lambda}_{i} R^{i(0)(2)}_{j\,k\,h} + \nabla^{(2)}_{h} F^{\lambda}_{jk}$

$+ \frac{2}{3} J^{\lambda}_{(2)j(2)a} S^{(0)(2)a}_{k\,h(1)}$

前頁脚註

$dF^{\lambda}_{jk} + W^{\lambda}_{\mu} F^{\mu}_{jk} - W^{a}_{j} F^{\lambda}_{ak} - W^{a}_{k} F^{\lambda}_{ja}$

$\nabla^{(0)}_{h} F^{\lambda}_{jk} \cdot dy^{h} + \nabla^{(1)}_{h} F^{\lambda}_{jk} \delta y^{(1)h} + \nabla^{(2)}_{h} F^{\lambda}_{jk} \delta y^{(2)h}$

又、(1.5) ノ積分可能條件ハ次ノ通リデアル。

(2, 2)

a) $$\overset{(0)(0)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{j}\xi^{\sigma}_{k}+\frac{1}{3}\left(\overset{(0)(1)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{j}\mathfrak{J}^{\sigma}_{(2)k}-\overset{(0)(1)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{k}\mathfrak{J}^{\sigma}_{(2)j}\right)$$

$$=\xi^{\lambda}_{i}\overset{(0)(0)i}{S}_{jk(0)}+F^{\lambda}_{jk}-F^{\lambda}_{kj}$$

b) $$\overset{(0)(1)\lambda}{S}_{\mu\sigma(0)}\xi^{\mu}_{j}\xi^{\sigma}_{k}=\xi^{\lambda}_{i}\overset{(0)(1)i}{S}_{jk(0)}+\frac{2}{3}\mathfrak{J}^{\lambda}_{(2)j(2)k}$$

c) $$\overset{(0)(0)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{j}\xi^{\sigma}_{k}+\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{j}\mathfrak{J}^{\sigma}_{(2)k}$$

$$-\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{k}\mathfrak{J}^{\sigma}_{(2)j}-\frac{1}{3}\overset{(0)(2)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{k}\overset{(1)}{\overline{\nabla}}_{j}\mathfrak{J}^{\sigma}$$

$$+\frac{1}{3}\overset{(0)(2)\lambda}{S}_{\mu\sigma(1)}\xi^{\mu}_{j}\overset{(1)}{\overline{\nabla}}_{k}\mathfrak{J}^{\sigma}=\xi^{\lambda}_{i}\overset{(0)(0)i}{S}_{jk(1)}$$

$$+\frac{1}{3}\mathfrak{J}^{\lambda}_{(2)i}\overset{(0)(0)i}{S}_{jk(0)}+\frac{1}{3}\overset{(0)}{\nabla}_{k}\mathfrak{J}^{\lambda}_{(2)j}-\frac{1}{3}\overset{(0)}{\nabla}_{j}\mathfrak{J}^{\lambda}_{(2)k}$$

d) $$\overset{(0)(0)\lambda}{S}_{\mu\sigma(2)}\xi^{\mu}_{j}\xi^{\sigma}_{k}+\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(2)}\xi^{\mu}_{j}\mathfrak{J}^{\sigma}_{(2)k}$$

$$-\frac{1}{3}\overset{(0)(1)\lambda}{S}_{\mu\sigma(2)}\xi^{\mu}_{k}\mathfrak{J}^{\sigma}_{(2)j}+\frac{1}{9}\overset{(1)(1)\lambda}{S}_{\mu\sigma(2)}\mathfrak{J}^{\mu}_{(2)j}\mathfrak{J}^{\sigma}_{(2)k}$$

$$=\xi^{\lambda}_{i}\overset{(0)(0)i}{S}_{jk(2)}+\frac{2}{3}\mathfrak{J}^{\lambda}_{(2)i}\overset{(0)(0)i}{S}_{jk(1)}+\frac{1}{3}(\overset{(1)}{\overline{\nabla}}_{i}\mathfrak{J}^{\lambda})\overset{(0)(0)i}{S}_{jk(0)}$$

$$+\frac{1}{3}\overset{(0)}{\nabla}_{k}\overset{(1)}{\overline{\nabla}}_{j}\mathfrak{J}^{\lambda}-\frac{1}{3}\overset{(0)}{\nabla}_{j}\overset{(1)}{\overline{\nabla}}_{k}\mathfrak{J}^{\lambda}$$

e) $\overset{(0)(1)\lambda}{S}_{\mu\sigma(2)}\xi^{\mu}_{j}\xi^{\sigma}_{k}+\frac{1}{3}\overset{(1)(1)\lambda}{S}_{\mu\sigma(2)}J^{\mu}_{(2)j}\xi^{\sigma}_{k}$

$$-\frac{1}{3}\overset{(1)(1)\lambda}{S}_{\mu\sigma(2)}J^{\mu}_{(2)k}\xi^{\sigma}_{j}=\xi^{\lambda}_{i}\overset{(0)(1)i}{S}_{jk(2)}-\frac{2}{3}\overset{(0)}{\nabla}_{j}J^{\lambda}_{(2)k}$$

$$+\frac{2}{3}J^{\lambda}_{(2)i}\overset{(0)(1)i}{S}_{jk(2)}+\frac{1}{3}\overset{(1)}{\nabla}_{k}\overset{(1)}{\nabla}_{j}J^{\lambda}+\frac{1}{3}\left(\overset{(1)}{\nabla}_{i}J^{\lambda}\right)\overset{(0)(1)i}{S}_{jk(0)}$$

コヽデ (2.1),(2.2) ノ諸式ノ意味ヲ考ヘナケレバナラヌ。先ヅ(0.8)ヲ見レバ

$\delta x^{(1)\lambda}=0,\ \delta x^{(2)\lambda}=0$ ノ條件ノ下ニ恒ニ

$\underset{2}{\delta}\underset{1}{\delta}v^{\lambda}-\underset{1}{\delta}\underset{2}{\delta}v^{\lambda}=0$ トナルタメニハ $\overset{\nu\lambda(0)(0)}{R}_{\mu\sigma\pi}=0$ ガ必要充分。

$\delta x^{(1)\lambda}=0,\ \delta x^{(2)\lambda}=0$ ノ條件ノ下ニ恒ニ

$\underset{2}{\delta}\underset{1}{\delta}v^{\lambda}-\underset{1}{\delta}\underset{2}{\delta}v^{\lambda}=0$ トナルタメニハ $\overset{\nu\lambda(1)(1)}{R}_{\mu\sigma\pi}=0$ ガ必要充分。

デアルカラ

$$\left.\begin{array}{lll} \overset{\nu\lambda(0)(0)}{R}_{\mu\sigma\pi}=0 & \text{ノヤウナ } \nabla_n \text{ ヲ} & 0\text{-}flat \text{ ナ } \nabla_n \\ R^{\lambda(1)(1)}_{\mu\sigma\pi}=0 & \quad〃 & I\text{-}flat \quad 〃 \end{array}\right\}$$

ト名付ケヤウ。

又(0.10)ヲ見ルニ

a) $\delta x^{(1)\sigma}=0$ トシテ恒ニ $\underset{2}{\delta}\underset{11}{d}x^{\lambda}-\underset{1}{\delta}\underset{2}{d}x^{\lambda}=0$ ト

ナルタメニハ $S^{(0)(0)\lambda}_{u\sigma(0)}=0$ ガ必要充分。

b) $\delta x^{(2)\sigma}=0$ トシテ恒ニ $\underset{2}{\delta}\underset{1}{\delta}x^{(1)\lambda}-\underset{1}{\delta}\underset{2}{\delta}x^{(1)\lambda}=0$ トナルタメニハ $S^{(0)(0)\lambda}_{u\sigma(1)}=0,\ S^{(0)(1)\lambda}_{u\sigma(1)}=0$ ガ必要充分。

c) $\delta x^{(1)\sigma}=0$ トシテ恒ニ $\underset{2}{\delta}\underset{1}{\delta}x^{(2)\lambda}-\underset{1}{\delta}\underset{2}{\delta}x^{(2)\lambda}=0$ トナルタメニハ $S^{(0)(0)\lambda}_{u\sigma(2)}=0$ ガ必要充分。

c) $dx^{\sigma}=0$ トシテ恒ニ $\underset{2}{\delta}\underset{1}{\delta}x^{(2)\lambda}-\underset{1}{\delta}\underset{2}{\delta}x^{(2)\lambda}=0$ トナルタメニハ $S^{(1)(1)\lambda}_{u\sigma(2)}=0$ ガ必要充分。

デ、$S^{(0)(1)\lambda}_{u\sigma(1)}=S^{(1)(1)\lambda}_{u\sigma(2)}=S^{(0)(0)\lambda}_{u\sigma(0)}$ ナルヲ以テ、3通リニ分類スレバヨイ。

$$\begin{cases} 0\text{-symmetric} \cdots\cdots S^{(0)(0)\lambda}_{u\sigma(0)}=0 \\ \mathrm{I}\text{-symmetric} \cdots\cdots S^{(0)(0)\lambda}_{u\sigma(1)}=0 \\ 2\text{-symmetric} \cdots\cdots S^{(0)(0)\lambda}_{u\sigma(2)}=0 \end{cases}$$

尤モ $R^{\lambda(0)(0)}_{\tau u\sigma}x^{(1)\tau}=S^{(0)(0)\lambda}_{u\sigma(1)},\quad \left(S^{(0)(0)\lambda}_{u\sigma(0)}\right)_{(2)\tau}=\frac{1}{2}R^{\lambda(1)(1)}_{\tau u\sigma}$

ノ如キ関係ガアルカラ

$$\left.\begin{matrix} 0\text{-flat} \longrightarrow \mathrm{I}\text{-symm} \\ 0\text{-symm} \longrightarrow \mathrm{I}\text{-flat} \end{matrix}\right\}$$

ナルコトガ分ル。ソコデ

(2.1.a) ヨリ

(A) <u>0-flat ナ V_n ノ中ノ totally geodesic V_{n-1} ハ</u>

<u>又 0-flat</u>

(2.1.b) ヨリ

(B) <u>I-flat ナ V_n ノ中ノ semi-geodesic V_{n-1} ハ</u>

<u>又 I-flat</u>

(2.2.a) ヨリ

(C) <u>0-symm. ナ V_n ノ中ノ semi-geodesic V_{n-1} ハ</u>

<u>又 0-symm</u>

(2.2.c) ヨリ

(D) <u>I-symm. ナ V_n ノ中ノ totally geodesic V_{n-1} ハ</u>

<u>又 I-symm</u>

ノヤウナコトガ見エル。(A)カラ(D)が，(C)カラ(B)が直接ニ出ル。

又(2.2.d)ヨリ

(E) <u>2-symm. ナ V_n ノ中ノ totally geodesic V_{n-1} ハ</u>

<u>又 2-symm</u>

ナルコトが知レル。

残サレタ問題ハ V_{n-1} ノパスヲ、如何ニシテ決定スルカト云フコトデアルが、コレハ別ノ機会ニ廻シテ、今ハ只コレダケデシテ置ク

終リニ自分勝手ナ、名称ヤ定義ヲ用ヒタ所が少クナイノデ此ノ点本部先生ニ御許シヲ乞フ次第デアル。又、矢野

健太郎先生ノ御指導ヲ感謝致シマス。

（昭和18年1月24日）