

# 546. 円系表面 = ツイテ

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(I) 円系表面上ノ極小曲線ハ

$$(1) (\theta_t \theta_\tau) dt^2 + 2(\theta_t \theta_c) dt d\tau + (\theta_c \theta_c) d\tau^2 = 0$$

テ表ハサレルコトヲ自分ハ前ニ述ベタ。

サテ、ソレヲ

$$(2) d\varphi^2 + d\psi^2 = 0$$

ニテ表ハシ得ベシ。ソレニハ  $\varphi$  ハ

$$(3) \frac{\partial}{\partial t} \left( \frac{(\theta_c \theta_c) \frac{\partial \varphi}{\partial t} - (\theta_t \theta_c) \frac{\partial \varphi}{\partial \tau}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} \right) + \frac{\partial}{\partial \tau} \left( \frac{(\theta_t \theta_t) \frac{\partial \varphi}{\partial \tau} - (\theta_t \theta_c) \frac{\partial \varphi}{\partial t}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} \right) = 0$$

ノ解デアアル。コノトキ

$$-\frac{(\theta_t \theta_t) \frac{\partial \varphi}{\partial t} - (\theta_t \theta_c) \frac{\partial \varphi}{\partial t}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} dt + \frac{(\theta_c \theta_c) \frac{\partial \varphi}{\partial t} - (\theta_t \theta_c) \frac{\partial \varphi}{\partial t}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} d\tau$$

ハ完全微分トナリ、ソレヲ  $d\psi$  トスル。此ノ  $\varphi, \psi$  ヲトレバ

(2) ノ如ク極小曲線ヲ表シヨベシ。

ツマリ、ソコニ

$$\left\{ \begin{array}{l} \frac{\partial \psi}{\partial t} = \frac{(\theta_t \theta_t) \frac{\partial \varphi}{\partial t} - (\theta_t \theta_c) \frac{\partial \varphi}{\partial t}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} \\ \frac{\partial \psi}{\partial \tau} = \frac{(\theta_c \theta_c) \frac{\partial \varphi}{\partial t} - (\theta_t \theta_c) \frac{\partial \varphi}{\partial t}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\partial \varphi}{\partial t} = \frac{(\theta_t \theta_t) \frac{\partial \psi}{\partial t} - (\theta_t \theta_c) \frac{\partial \psi}{\partial t}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} \\ \frac{\partial \varphi}{\partial \tau} = \frac{-(\theta_t \theta_c) \frac{\partial \psi}{\partial t} - (\theta_t \theta_c) \frac{\partial \psi}{\partial \tau}}{\sqrt{(\theta_t \theta_t)(\theta_c \theta_c) - (\theta_t \theta_c)^2}} \end{array} \right.$$

デアアル。

(II) 円素表面ノ法曲率ヲ  $\frac{1}{R}$  トセバ

$$(1) \frac{1}{R} = \frac{\lambda [L dt^2 + 2M dt d\tau + N d\tau^2]}{(\theta_t \theta_t) dt^2 + 2(\theta_t \theta_c) dt d\tau + (\theta_c \theta_c) d\tau^2}$$

デアアル。記法ハ前々ノ通りデアアル。入ノ決定方ニツイテハ以

前ニノベタカラ其様ニ定メルモノトスル。(1)カラ

$$(2) \quad -\frac{1}{R} \{ (\theta_t \theta_t) dt^2 + 2(\theta_t \theta_\tau) dt d\tau + (\theta_\tau \theta_\tau) d\tau^2 \} \\ + L dt^2 + 2M dt d\tau + N d\tau^2 = 0$$

(2) 7  $dt = \psi \delta \tau$ 、マタ  $d\tau = \psi \delta \tau$  微分シ、 $\frac{1}{R}$ 、極大極小 = 對スル條件ヲ求メ Rヲ消去セバ

$$(3) \quad \begin{vmatrix} (\theta_t \theta_t) dt + (\theta_t \theta_\tau) d\tau & L dt + M d\tau \\ (\theta_\tau \theta_t) dt + (\theta_\tau \theta_\tau) d\tau & M dt + N d\tau \end{vmatrix} = 0$$

ヲ得。但シ  $\lambda \neq 0$  トス。

(3) ハ  $\frac{1}{R}$ 、極大及ビ極小 = 對スル  $dt : d\tau$  方向ヲ與フル二次方程式トナリ、ツマリ曲率曲線ノ方程式デアリ。

(III) M. Chini ノ論文 (*Sulla determinazione delle geodetiche di talune superficie*, *Rendiconti Istituto Lombardo* (2) 59, 298-302) ヨリ吾々ノ円系表面ノ基本量  $(\theta_t \theta_t)$ ,  $(\theta_t \theta_\tau)$ ,  $(\theta_\tau \theta_\tau)$ 、間

$$(\theta_t \theta_t) = 1, \quad (\theta_\tau \theta_\tau) = 0,$$

$$(\theta_t \theta_\tau) = \frac{1}{\{t + f(\tau)\}^2} \dots \dots (1)$$

ガ成立ツ場合 = 其上ノ測地線ノ一般方程式ハ

$$\frac{(\theta_t \theta_t) \sqrt{\sin \alpha}}{\sin \tau} - \int \frac{f(\tau) \sqrt{\sin \alpha}}{\sin \tau} d\tau = \text{const.} \quad (2)$$

ヲ導ヘラルルコトガ可カル。

但シ  $\alpha$  ハ

$$\sqrt{\sin \alpha} \cot \tau + \frac{1}{2} \int \sqrt{\sin \alpha} d\alpha = C \dots \dots (3)$$

ヲ満足スル。コトニシテハ帯數ヲアル。